UDAAN

PRELIMS WALLAH (STATIC)

PRELIMS 2025

CSAT

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**QUICK AND COMPREHENSIVE REVISION
SERIES FOR PRELIMS 2025**

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PREFACE

A highly skilled professional team of PW ONLY IAS works arduously to ensure that the students receive the best content for the UPSC exam.

A plethora of UPSC study materials are available in the market, but PW ONLY IAS professionals continuously work to provide supreme-quality study material for our UPSC students.

From the beginning, the whole content team comprising Content Creators, Reviewers, DTP operators, Proofreaders and others are involved in shaping the material to their best knowledge and experience to produce powerful content for the students.

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PW ONLY IAS strongly believes in conceptual and fun-based learning and provide highly exam-oriented content to bring quality and clarity to the students.

This book adopts a multi-faceted approach to mastering and understanding the concepts and equipping the students with the knowledge for this competitive exam.

The main objective of the study material is to provide short, crisp, concise and high-quality content to our students.

BOOK FEATURES

- Covers all important topics that are Frequently asked in CSAT.
- Highlights different types of questions through important PYQs.
- Provides all the important formulas and easy-to-remember Tricks.
- Detailed Explanations to solve different type of question.

CONTENTS

Quantitative Aptitude

Number System

TYPE OF NUMBERS

A number system is a systematic way of expressing and representing numbers. It includes a set of symbols, rules, and conventions for counting and performing arithmetic operations.

Real Numbers

Rational
$$\left\{-8, \frac{-12}{6}, 0, \frac{8}{6}\right\}$$

Irrational
$$\left\{\sqrt{2}, \sqrt{3}, \sqrt{4}\right\}$$

Integers
$\{-3, -2, -1, 0, 1, 2, 3...\}$

Fractions & Mixed Numbers
$$\left\{\frac{1}{4}, 4\frac{7}{12}, 3\frac{1}{9}\right\}$$

Whole Numbers
$\{0, 1, 2, 3, 4, 5, 6...\}$

Negative Integers
$\{-5, -4, -3, -2, -1.\}$

Natural Numbers
$\{1, 2, 3, 4, 5, 6...\}$

Zero

Even Numbers
$\{2, 4, 6, 8, 12, 28\}$

Odd Numbers
$\{1, 3, 5, 9, 27, 33\}$

Prime Numbers
2

Composite Numbers
$\{6, 10, 12, 16\}$

Prime Numbers
$\{3, 5, 7, 11, 13...\}$

Composite Numbers
$\{9, 15, 27, 33...\}$
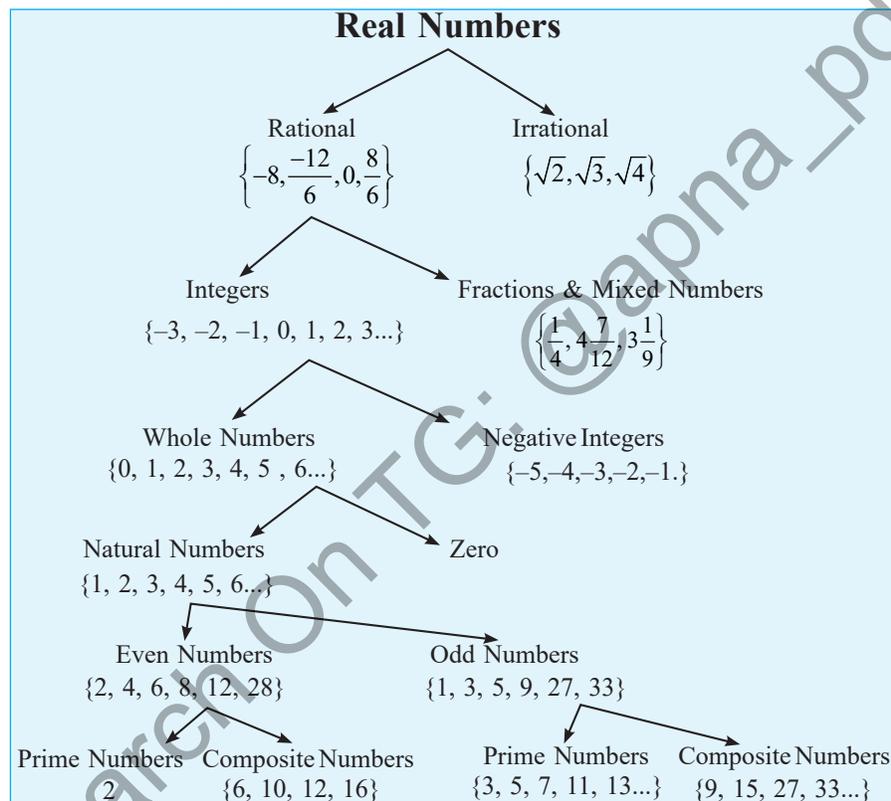
Questions based on properties of even and odd numbers:

1. Let p, q, r and s be distinct positive integers. Let p, q be odd and r, s be even. Consider the following statements: **(UPSC CSAT 2024)**

1. $(p - r)^2 (qs)$ is even.

2. $(q - s)q^2s$ is even.

3. $(q + r)^2 (p + s)$ is odd.

Which of the statements given above are correct?

(*a*) 1 and 2 only

(*b*) 2 and 3 only

(*c*) 1 and 3 only

(*d*) 1, 2 and 3

Sol: To solve this question we need to know about the following properties of even/odd numbers:

(i) The result is always an even number in the case of the following mathematical operations-

Even $\pm$ Even = Even

Even $\times$ Even/Odd = Even

(Even)$^{\text{Even or Odd}}$ = Even.

Odd $\pm$ Odd = Even

(ii) The result will always be an odd number in the case of the following:

Even $\pm$ Odd = Odd

Odd $\times$ Odd = Odd

(Odd)$^{\text{Even or Odd}}$ = Odd.

Now, In the question it is given that p and q are odd and r and s are even.

For Statement 1: $(p - r)^2 (qs)$

Here 'p' is odd and 'r' is even and by the properties,

Even $\pm$ odd = odd and (Odd)$^{\text{Even or Odd}}$ = odd.

So, $(p - r)^2$ is odd.

Now, 'q' is odd, and s is even, according to the properties discussed above:

Even $\times$ (Even or odd) = Even.

So, (qs) is even

And $(p - r)^2 (qs)$ is even $\times$ odd = even.

So, statement 1 is correct.

For Statement 2:

Since, 's' is even.

So, $(q - s)(q^2 s)$ is even because even or odd multiple of even is always even.

So, statement 2 is correct.

For Statement '3':

Since, $(p + s) = $ (odd + even) = odd

Also, $(q + r) = $ (odd + even) = odd

Thus, $(q + r)^2 (p + s)$ is odd

So, statement 3 is correct.

Hence, option (d) is correct.

A similar question was asked in 2023 also:

2. **Three of the five positive integers 'p', 'q', 'r', 's', and 't' are even and two of them are odd (not necessarily in order). Consider the following:** (UPSC CSAT 2023)

   I. $(p + q + r - s - t)$ is definitely even.

   II. $(2p + q + 2r - 2s + t)$ is definitely odd.

   Which of the above statements is/are correct?

   (a) I Only
   (b) II Only
   (c) Both I and II
   (d) Neither I nor III

Sol: For '**I**': $p + q + r - s - t = p + q + r - (s + t)$

Case (i): If all 'p', 'q', and 'r' are even and both 's' and 't' are odd, then $(p + q + r)$ will be even while $(s + t)$ will be even. Thus even – even = even

Case (ii): If one among 'p', 'q', and 'r' is odd while one of 's' and 't' is odd, then both $(p + q + r)$ & $(s + t)$ will be odd. Thus odd – odd = even

Case (iii): If two among 'p', 'q', and 'r' are odd while both 's' and 't' are even, then both $(p + q + r)$ & $(s + t)$ will be even. Thus even – even = even

For '**II**': $2p + q + 2r - 2s + t = (2p + 2r - 2s) + (q + t)$

Here, $(2p + 2r - 2s)$ will always be even irrespective of the nature of 'p', 'q', and 'r'

Now, if both 'q' and 't' are even/odd then the overall result will be even.

But, if any one of 'q' and 't' is odd, then the overall result will be odd.

Thus, $2p + q + 2r - 2s + t$ can be even or odd depending upon the nature of 'q' and 't'.

Therefore, only 'I' is true.

Hence, option (a) is correct.

3. **Consider the following in respect of prime number 'p' and composite number 'c'.** (UPSC CSAT 2023)

   I. $\dfrac{(p + c)}{(p - c)}$ can be Even

   II. $(2p + c)$ can be odd.

   III. 'pc' can be odd.

   Which of the statements given above are correct?

   (a) I and II only
   (b) II and III only
   (c) I and III only
   (d) I, II and III

Sol: Now the prime numbers are the numbers that are divisible by either '1' or by the number itself and the composite numbers are the numbers that are divisible by at least 3 numbers ('1', the number itself, and one more number).

NOTE: '2' is the only prime number which is even i.e. all the even numbers except '2' are composite numbers.

The solution to the question: Since in all three conclusions we are asked 'can be' i.e. possibility. Thus, in these questions, we should go through the 'Hit & Trial' method to prove that the given statement is true at least once.

Let's substitute 'p' and 'c' values with 11 and 9.

Statement I:

$\dfrac{p + c}{p - c} = \dfrac{11 + 9}{11 - 9} = \dfrac{20}{2} = 10$ (even)

So, statement I is correct.

Statement II:

$2p + c = (2 \times 11) + 9 = 31$ (an odd number)

So, statement II is correct.

Statement III:

pc = $11 \times 9 = 99$ (an odd number)

So, statement III is correct.

Thus, all the conclusions are true.

Hence, option (d) is correct.

PLACE VALUE AND FACE VALUE

Face Value: The face value of a digit in a numeral is the actual value of that digit, regardless of its position within the numeral. To put it simply, it's the inherent worth of the digit itself.

In the number 28746, the face value of:

The place value of 8 is 8

The place value of 7 is 7

The place value of 6 is 6

The place value of 4 is 4

Place Value (or Local Value): The place value is a value assigned to a digit based on its position within the number. The place value of a digit changes as its position changes within the number..

For example: In the number 28746

The place value of 7, in the hundreds place, is 700.

The place value of 4, in the tens place, is 40.

The place value of 6, in the ones place, is 6.

Ten thousands	Thousands	Hundreds	Tens	Ones
× 10000	× 1000	× 100	× 10	× 1
2	8	7	4	6

So, from this we can conclude that 28746

$= 2 × (10,000) + 8 × (1000) + 7 × (100) + 4 × 10 + 6.$

Deductions:

Any two digit number "**ab**" can be written as: **10a + b.** Any three digit number "**abc**" can be written as **100a + 10b + c** and so on.

To understand this concepts lets solve some questions previously asked by UPSC:

1. **Let X be a two-digit number and Y be another two-digit number formed by interchanging the digits of X. If (X + Y) is the greatest two-digit number, then what is the number of possible values of X?**

   (UPSC CSAT 2024)

   (*a*) 2   (*b*) 4

   (*c*) 6   (*d*) 8

Sol: Let 'X' = $10a + b$

So, Y = $10b + a$

Now, X + Y = $(10a + b) + (10b + a) = (11a + 11b)$

According to question;

X + Y = greatest two digit number = 99

Or, $11(a + b) = 99$

So, $(a + b) = 9$

Possible values of (a, b) = (1, 8), (2, 7), (3, 6), (4. 5), (5,4), (6, 3), (7, 2) and (8, 1) i.e. 8

Hence, option (d) is correct.

2. **A 3-digit number ABC, on multiplication with D, gives 37DD where 'A', 'B', 'C' and 'D' are different non-zero digits. What is the value of (A + B + C)?**

   (UPSC CSAT 2023)

   (*a*) 18

   (*b*) 16

   (*c*) 15

   (*d*) Cannot be determined due to insufficient data

Sol: It's given that, ABC × D = 37DD, Where A, B, C, and D are different non zero digits.

ABC = $\dfrac{37DD}{D} = \dfrac{3700 + 10D + D}{D}$ (we added the place value of both "D" in 37DD)

ABC = $\dfrac{3700}{D} + 11$

Now ABC is an integer so $\dfrac{3700}{D}$ should also be an integer. So, the values of "D" can be 1, 2, 4, and 5.

If D = 1, ABC = 3700 + 11 = 3711 which is not possible because ABC is a three-digit number.

If D = 2, ABC = 1850 + 11 = 1861 which is not possible because ABC is a three-digit number.

If D = 4, ABC = 925 + 11 = 936

So, D = 4 is possible and ABC = 936.

So, **A + B + C = 9 + 3 + 6 = 18.**

If D = 5, ABC = 740 + 11 = 751 which is not possible because B and D cannot be the same.

Hence, option (a) is correct.

DIVISIBILITY

Divisibility By	Condition
2	If the unit's digit of a number is 0, 2, 4, 6 or 8.
3	If the sum of the digits of a number is divisible by 3.
4	If the last 2 digits of a number is divisible by 4 or the last 2 digits of a number are zero.
5	If the last digit of a number is either 0 or 5.
6	If the number is divisible by 2 and 3 both.
7	If the difference between twice the last digit and the number formed by the remaining digits is either zero or a multiple of 7.
8	If the last three digits of a number is divisible by 8 or the last three digits of a number are zero.
9	If the sum of the digits of a number is divisible by 9.
11	If the difference between the sum of its digits at odd places and the sum of its digits at even places is either 0 or a multiple of 11.
12	If the number is divisible by 3 as well as 4.

To understand this concepts lets solve some questions previously asked by UPSC:

1. **If R and S are different integers both divisible by 5, then which of the following is not necessarily true?**

   (UPSC CSAT 2016)

   (*a*) R – S is divisible by 5 (*b*) R + S is divisible by 10

   (*c*) RS is divisible by 25   (*d*) $R^2 + S^2$ is divisible by 5

Sol: We will understand about the properties of division through this question.

Property I: If 'a' and 'b' are divisible by 'n' then $a \pm b$ is also divisible by 'n'.

So in this question, R and S are divisible by 5 then R ± S is also divisible by 5.

So, option (a) is correct.

Property II: If 'a' and 'b' are divisible by 'n' then "ab" is divisible by n^2.

So in this question, R and S are divisible by 5 then RS will be divisible by $5^2 = 25$.

So, option (c) is correct.

Property III: If "a" is divisible by "n" then a^m will be divisible by "n" for any natural number "m".

So, in this question R and S are divisible by 5 hence R^2 and S^2 will be divisible by 5 and then by property I, $R^2 + S^2$ is divisible by 5.

So, option (d) is correct.

Option (b) is not correct because if R = 10 and S = 5 then, R + S = 10 + 5 = 15, which is not divisible by 10.

Hence, option (b) is correct.

2. **For any choices of values of 'X', 'Y' and 'Z', the 6 digit number of the form XYZXYZ is divisible by:**
   (UPSC CSAT 2023)

   (*a*) 7 and 11 only   (*b*) 11 and 13 only

   (*c*) 7 and 13 only   (*d*) 7, 11 and 13

Sol: Given 6 digit number is "XYZXYZ"

As we know from place value/face value, the number can be written as:

XYZXYZ = 100000 × X + 10000 × Y + 1000 × Z + 100 × X + 10 × Y + Z = 100100 × X + 10010 × Y + 1001 × Z = 1001(100X + 10Y + Z)

Now 1001 = 7 × 11 × 13,

And, XYZXYZ = 7 × 11 × 13 (100X + 10Y + Z)

So, the number is divisible by 7, 11 and 13.

Hence, option (d) is correct.

CYCLICITY AND UNIT DIGIT

The cyclical behavior of numbers depends upon their units (ones) digits. Each unit's digit shows a repetitive pattern which depends upon their powers.

For Example: Understanding the Cyclicity of 7

$7^1 = 7$ (Unit Digit = 7), $7^2 = 49$ (Unit Digit = 9)

$7^3 = 343$ (Unit Digit = 3), $7^4 = 2401$ (Unit Digit = 1)

$7^5 = 16807$ (Unit Digit = 7)

Hence, Number "7" has a cyclicity of 4 and

Power Cycle (Pattern of last digits) of 7, 9, 3, 1.

Cyclicity cycle can be summarized as shown in the table below:

Number	Cyclicity	Power cycle
0	1	0
1	1	1
2	4	2, 4, 8, 6
3	4	3, 9, 7, 1
4	2	4,6
5	1	5
6	1	6
7	4	7, 9, 3, 1
8	4	8, 4, 2, 6
9	2	9, 1

Methods to Find Unit Digit

1. If the unit digit of any number is 0, 1, 5 or 6, then the unit's digit remains the same, irrespective of the power.

2. If the unit's digit is 2, 3, 4, 7, 8 and 9.

   Step1: Divide the power by 4 and find the remainder.

   Step2: Apply cyclicity rule on the power on unit digit by using the remainder

To understand this concepts lets solve some questions previously asked by UPSC:

1. **What is the unit digit in the expansion of $(57242)^{9 \times 7 \times 5 \times 3 \times 1}$?**
   (UPSC CSAT 2023)

   (*a*) 2   (*b*) 4

   (*c*) 6   (*d*) 8

Sol: **Note:** The unit digit of the resultant number only depends on the unit digit of the given number 57242, i.e. Unit digit of $(57242)^{9 \times 7 \times 5 \times 3 \times 1}$

= unit digit of $(2)^{9 \times 7 \times 5 \times 3 \times 1}$

Now, we know that: $2^1 = 2$, $2^2 = 4$, $2^3 = 8$, $2^4 = 16$ (unit digit 6), $2^5 = 32$ (unit digit 2) and so on.

So, the cyclicity of '2' is 4.

Exponent of any number ending with 2 will produce a number that will end in 2, 4, 8, or 6.

$9 \times 7 \times 5 \times 3 \times 1 = 945 = 4 \times 236 + 1$

So, unit digit of $(2)^{9 \times 7 \times 5 \times 3 \times 1}$ = unit digit of $2^{4 \times 236 + 1}$ = unit digit of $2^1 = 2$. (cyclicity of 2 is 4)

Hence, option (a) is correct.

2. **If 3^{2019} is divided by 10, then what is the remainder?**
   (UPSC CSAT 2021)

   (*a*) 1   (*b*) 3

   (*c*) 7   (*d*) 9

Sol: Note: Remainder when a number is divided by 10 = unit digit of the number.

So, we need to find the unit digit of 3^{2019}

We know the cyclicity of 3 is 4 and 2019 = 4 × 504 + 3.

So, the unit digit of 3^{2019} = Unit digit of 3^3

= unit digit of 27 = 7.

Thus, the required remainder is 7.

Hence, option (c) is correct.

REMAINDER

Dividend = (Divisor × Quotient) + Remainder

where,

- Dividend = The number which is being divided
- Divisor = The number by which division process is performed
- Quotient = Greatest possible integer obtained as a result of division
- Remainder = Left part of dividend which cannot be further divided by the divisor.

Properties of Remainder:

1. Remainder of $\dfrac{a+b}{n}$ = Remainder of $\left(\dfrac{a}{n}\right)$ + Remainder of $\left(\dfrac{b}{n}\right)$ but if the remainder is more than n, it will be further divided by n.

2. The remainder of multiplication of a number by a value is the same as multiplication of the remainder of individual terms.

   Remainder of $\left(\dfrac{a \times b}{n}\right)$ = Remainder of $\left(\dfrac{a}{n}\right)$ × Remainder of $\left(\dfrac{b}{n}\right)$ but if the remainder is more than n, it will be further divided by n.

3. Fermat's Theorem: If the given equation can be expressed as $\dfrac{a^{(P-1)}}{P}$ then the remainder is 1, where P is prime number and a, P are co-prime.

To understand this concepts lets solve some questions previously asked by UPSC:

1. **What is the remainder if 2^{192} is divided by 6?**

   (UPSC CSAT 2023)

   (a) 0   (b) 1

   (c) 2   (d) 4

Sol: In this question as well we will observe a pattern:

2^1 divided by 6 leaves a remainder of 2.

2^2 divided by 6 leaves a remainder of 4.

2^3 divided by 6 leaves a remainder of 2.

2^4 divided by 6 leaves a remainder of 4.

2^5 divided by 6 leaves a remainder of 2.

We notice that when power of 2 is odd, the remainder is 2, and, when power of 2 is even, the remainder is 4. Therefore, the answer will be 4.

Hence, option (d) is correct.

Note: the remainder of $\dfrac{2^{192}}{6} \neq$ remainder of $\dfrac{2^{191}}{3}$, though both the fractions are equivalent.

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2. **A number 'N' is formed by writing 9 for 99 times. What is the remainder if 'N' is divided by 13?**

   (UPSC CSAT 2023)

   (a) 11   (b) 9

   (c) 7   (d) 1

Sol: In this type of questions we need to analyse the pattern of remainders.

So, the pattern in this question is:

Remainder of $\left[\dfrac{9}{13}\right]$ = 9	Remainder of $\left[\dfrac{99}{13}\right]$ = 8
Remainder of $\left[\dfrac{999}{13}\right]$ = 11	Remainder of $\left[\dfrac{9999}{13}\right]$ = 2
Remainder of $\left[\dfrac{99999}{13}\right]$ = 3	Remainder of $\left[\dfrac{999999}{13}\right]$ = 0

This pattern can be seen getting repeated thereafter too.

Remainder of $\left[\dfrac{9999999}{13}\right]$ = 9

Remainder of $\left[\dfrac{99999999}{13}\right]$ = 8 …and so forth.

And, if the total number of 9's is six, twelve, eighteen, ……., ninety, ninety six, etc., the remainder is 0.

So, if the number has ninety seven 9's, the remainder is 9. [Following the pattern]

And, if the number has ninety eight 9's, the remainder is 8.

So, if the number has ninety nine 9's, the remainder is 11. Therefore, the answer is 11.

Hence, option (a) is correct.

Note: A number of the form 'aaaaaa' (a number formed by repeating a digit six times) is always divisible by 7, 11, 13, and 37.

NUMBER OF ZEROS

We will understand this concept through a question:

1. **How many consecutive zeros are there at the end of the integer obtained in the product $1^2 \times 2^4 \times 3^6 \times 4^8 \times ….. \times 25^{50}$?** **(UPSC CSAT 2023)**

   (a) 50   (b) 55

   (c) 100   (d) 200

Sol: The number of zeros at the end = number of factors of "10"

A pair of 2 and 5 makes a factor of 10.

So to count the number of zeros at the end, we need to calculate the number of pairs of 2's and 5's

Now the expression given in the question is:
$1^2 \times 2^4 \times 3^6 \times 4^8 \times ….. \times 25^{50}$

We need to calculate the 2's and 5's in this expression.

Every 2nd term of this expression contains an even number (multiple of 2), contributing to the number of 2's. So, clearly number of 2's will be more than the number of 5's in this expression.

So, the terms that will give us fives are:

5^{10}, 10^{20}, 15^{30}, 20^{40} and 25^{60}

5^{10} will give 10 fives.

$10^{20} = 2^{20} \times 5^{20}$ will give 20 fives

$15^{30} = 3^{30} \times 5^{30}$ will give 30 fives

$20^{40} = 4^{40} \times 5^{40}$ will give 40 fives.

And, $25^{50} = (5^2)^{50} = 5^{100}$ i.e. it will give 100 fives

So, total number of fives = $10 + 20 + 30 + 40 + 100 = 200$

So the total pairs of 2's and 5's are 200.

So, the number of zeros at the end = 200.

This is the process we need to follow to calculate the number of zeros at the end.

Hence, option (d) is correct.

SURDS & INDICES

Laws of Surds	Laws of Indices
1. $\sqrt[n]{a} = a^{\frac{1}{n}}$	1. $n^a \times n^b = n^{(a+b)}$
2. $\sqrt[m]{ab} = \sqrt[m]{a} \times \sqrt[m]{b} = a^{\frac{1}{m}} \times b^{\frac{1}{m}} = (ab)^{\frac{1}{m}}$	2. $\dfrac{n^a}{n^b} = n^{(a-b)}$
3. $\sqrt[m]{\dfrac{a}{b}} = \dfrac{\sqrt[m]{a}}{\sqrt[m]{b}} = \left(\dfrac{a}{b}\right)^{\frac{1}{m}}$	3. $(n^a)^b = n^{ab}$
4. $(\sqrt[n]{a})^m = a^{\frac{m}{n}} = \sqrt[n]{a^m}$	4. $(ab)^m = a^m \times b^m$
5. $\sqrt[m]{\sqrt[n]{a}} = \sqrt[m]{a^{\frac{1}{n}}} = a^{\frac{1}{mn}}$	5. $n^0 = 1$

Let us understand these laws through Some questions:

1. **Which number amongst 2^{40}, 3^{21}, 4^{18} and 8^{12} is the smallest?** (UPSC CSAT 2022)

   (a) 2^{40} (b) 3^{21}

   (c) 4^{18} (d) 8^{12}

Sol: We have to compare the numbers so let us try to rewrite the numbers with same bases for easier comparison

Thus, we can write them as:

$4^{18} = (2^2)^{18} = 2^{36}$ (as $4 = 2^2$)

$8^{12} = (2^3)^{12} = 2^{36}$ (as $8 = 2^3$)

From the above we can see that both $4^{18} = 8^{12}$.

Since, 2^{40} is greater than 2^{36}. So, 2^{40} cannot be the answer. Thus, we need to find the smallest one among 2^{36} and 3^{21}. As we cannot have two correct answers, the answer must be 3^{21}.

Hence, option (b) is correct.

2. **$15 \times 14 \times 13 \ldots 3 \times 2 \times 1 = 3^m \times n$. Where '$m$' and '$n$' are positive integers, then what is the maximum value of 'm'?** (UPSC CSAT 2022)

   (a) 7

   (b) 6

   (c) 5

   (d) 4

Sol: $15 \times 14 \times 13 \times 12 \times 11 \times 10 \times 9 \times 8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1 = 3^m \times n$

Numbers which are multiple of '3' in the expression written on LHS

$= 15 \times 12 \times 9 \times 6 \times 3$

$= (3 \times 5) \times (3 \times 4) \times (3 \times 3) \times (3 \times 2) \times 3 = 3^6 \times (5 \times 4 \times 2)$

Thus, the expression can be written as

$3^6 \times (5 \times 4 \times 2) \times 14 \times 13 \times 11 \times 10 \times 8 \times 7 \times 5 \times 4 \times 2 \times 1 = 3^m \times n$

On comparing both sides, we get; $m = 6$

Therefore, the maximum value of 'm' is 6.

Hence, option (b) is correct.

Some Important Formulae for the natural numbers:

(i) Sum of first 'n' natural numbers = $1 + 2 + 3 + \ldots + n$

$= \dfrac{n(n+1)}{2}$

(ii) Sum of square on first 'n' natural numbers

$= \dfrac{n(n+1)(2n+1)}{6}$

(iii) Sum of cubes of first 'n' natural numbers $= \left[\dfrac{n(n+1)}{2}\right]^2$

(iv) Sum of first 'n' even numbers = $2 + 4 + 6 + 8 + \ldots$upto 'n' terms = $n(n+1)$

(v) Sum of first 'n' odd numbers = n^2

Questions based on above formulae:

1. **Find the sum of the squares of the first 65 natural numbers.**

   (a) 91,665

   (b) 92,665

   (c) 93,665

   (d) 94,665

Sol: Sum of the square of first n natural numbers

$= \dfrac{n(n+1)(2n+1)}{6}$

Given, $n = 65$

$\therefore$ Required sum = $\dfrac{(65 \times 66 \times 131)}{6}$

$= 65 \times 11 \times 131 = 93,665$

Hence, option (c) is correct.

2. Find the difference between the sum of the cube of the first ten natural numbers and the sum of the first twenty natural numbers.

   (*a*) 2210   (*b*) 2625

   (*c*) 2815   (*d*) 2830

Sol: Sum of first '*n*' natural numbers = $\dfrac{n(n+1)}{2}$

n = 20 (given)

Sum of first 20 natural numbers = $\dfrac{(20 \times 21)}{2} = 10 \times 21 = 210$

Sum of cube of first '*n*' natural numbers = $\left[\dfrac{n(n+1)}{2}\right]^2$

n = 10 (given)

Sum of cube of first 10 natural numbers

$= \left[\dfrac{n(n+1)}{2}\right]^2 = \left[\dfrac{10(11)}{2}\right]^2 = (55)^2 = 3025$

Required difference = 3025 – 210 = 2815

Hence, option (c) is correct.

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H.C.F and L.C.M

HIGHEST COMMON FACTOR (H.C.F)

The highest number that exactly divides two or more given numbers leaving zero remainder is known as Highest Common Factor (HCF) of the given numbers.

Example: HCF of 24 and 36, is "12" as it is the highest number that divides 24 and 36 completely.

METHOD TO FIND HCF

Prime Factorisation Method.

Step 1: Write the given numbers in the form of the product of their prime factors.

Step 2: Choose all common prime factors with the lowest power of each common prime factor in the given numbers.

Step 3: Product of all common prime factors with their respective lowest power is the HCF.

For Example: Find the HCF of 729, 828 and 5625:

Prime Factorisation of:

$$729 = 3^6$$
$$828 = 2^2 \times 3^2 \times 23$$
$$5625 = 3^2 \times 5^4.$$
Therefore, HCF $= 3^2 = 9$.

Properties of HCF

- The HCF of two or more numbers is smaller than or equal to the smallest number of given numbers.
- Let "K" be the largest number which when divide a,b,c gives the same remainder 'r' & quotients are x, y, z respectively then **K = HCF of $(a - b, b - c, c - a)$**

  For example: What is the greatest number which leaves the same remainder in each case when divides 265, 580, and 825

  Explanation: Difference of the numbers
  $580 - 265 = 315$, $825 - 580 = 245$
  and $825 - 265 = 560$
  Factor of $315 = 3 \times 3 \times 5 \times 7$
  Factor of $245 = 5 \times 7 \times 7$
  Factor of $560 = 2 \times 2 \times 2 \times 2 \times 5 \times 7$
  Hence, HCF. of 315, 245, 560 $= 5 \times 7 = 35$

- The greatest number which divides x, y and z to leave the remainder R is;
  HCF of $(x - R)$, $(y - R)$ and $(z - R)$.

For Example: The greatest number which divides 16, 42 and 55 to leave the remainder 3 is equal to:
Given numbers: 16, 42 and 55
Remainder $= 3$
HCF of $(16 - 3)$, $(42 - 3)$, $(55 - 3)$ = HCF of 13, 39, 52
HCF of 13, 39, 52 $= 13$
Therefore, HCF of 16, 42, 55 is 13.

- The greatest number which divides x, y, z to leave remainders a, b, c is
  HCF of $(x - a)$, $(y - b)$ and $(z - c)$

  For Example: Calculate the greatest number which divides 17, 19, 21 to leave remainder 2, 4, and 6 respectively.
  Given numbers: 17, 19 and 21
  Respective remainder = 2, 4 and 6
  HCF of $(17 - 2)$, $(19 - 4)$, $(21 - 6)$ = HCF of 15, 15, 15
  HCF of 15, 15, 15 $= 15$
  Therefore, the greatest number is 15.

Lets understand these concepts of H.C.F with the help of questions:

1. **A can X contains 399 litres of petrol and a can Y contains 532 litres of diesel. They are to be bottled in bottles of equal size so that whole of petrol and diesel would be separately bottled. The bottle capacity in terms of litres is an integer. How many different bottle sizes are possible?** **(UPSC CSAT 2024)**

   (*a*) 3 (*b*) 4
   (*c*) 5 (*d*) 6

Sol: We have two containers with volume 399 litres and 532 liters that need to be bottled in bottles of equal size so that whole of petrol and diesel would be separately bottled. So that the volume of the bottle should be H.C.F of the 399 and 532.
   $399 = 3 \times 7 \times 19$ $532 = 2 \times 2 \times 7 \times 19$
   So, H.C.F (399, 532) $= 7 \times 19 = 133$.
   Now we can bottle the petrol and diesel in bottles of volume = factors of 133
   Factors of 133 = 1, 7, 19, 133.
   So, 4 sizes are possible.
   Hence, option (b) is correct.

2. **The greatest number that will divide 137, 345 and 770 leaving remainders 5, 9 and 2 respectively is**

   (*a*) 9

   (*b*) 5

   (*c*) 12

   (*d*) 14

Sol: By the property of H.C.F: The greatest number which divides *x*, y, z to leave remainders a, b, c is HCF of (*x* – *a*), (*y* – *b*) and (*z* – *c*).

Required number = HCF of (137 – 5), (345 –9) and (782 – 14). = HCF of 132, 336 and 768

Prime factorization of:

132 = 2 × 2 × 3 × 11   336 = 2 × 2 × 2 × 2 × 3 × 7

768 = 2 × 2 × 2 × 2 × 2 × 2 × 2 × 2 × 3

HCF of 132, 336 and 768 = 2 × 2 × 3 = 12

Hence, option (c) is correct.

LEAST COMMON MULTIPLE (L.C.M)

The LCM of numbers is the least possible number that is completely divisible by each of the numbers. For example, 15 is least possible number which is divisible by 3 and 5. Then the LCM of 3 and 5 is 15.

Method to find L.C.M: the best method to find L.C.M is the prime factorization method:

Step 1: Write the given numbers as a product of their prime factors.

Step 2: Choose the prime factors with the highest index in all the given numbers.

Step 3: Multiply all the prime factors with the respective highest indices to get the LCM of given numbers.

Example: LCM of 48, 76, 162.

Prime Factorisation of:

$48 = 2^4 \times 3$, $76 = 2^2 \times 19$, $162 = 2 \times 3^4$

$LCM = 2^4 \times 3^4 \times 19 = 24624$.

Properties of LCM

- The smallest number which is exactly divisible by *x*, *y* and *z* is LCM of *x*, *y*, *z*.
- The LCM of two or more numbers is greater than or equal to the greatest number of given numbers.
- The numbers which when divided by *x*, *y* and *z* leaves a remainder R in each case.

  Then the required numbers = (LCM of *x*, *y*, *z*) × K + R, where K is a constant
- The numbers which when divided by x, y and z leaves the remainder of a, b, c such that common difference (d) = x – a = y – b = z – c.

  Then the required numbers = (LCM of *x*, *y* and *z*) × K – d, where K is a constant

Lets understand the concept of L.C.M with the help of questions:

1. **There are three traffic signals. Each signal changes colour from green to red and then from red to green. The first signal takes 25 seconds, the second signal takes 39 seconds and the third signal takes 60 seconds to change the colour from green to red. The durations for green and red colours are the same. At 2:00 p.m, they turn green together. At what time will they change to green next, simultaneously?**

   (*a*) 4:00 p.m.   (*b*) 4:10 p.m.

   (*c*) 4:20 p.m.   (*d*) 4:30 p.m.

Sol: The three traffic signals turn from green to red at an interval of 25 seconds, 39 seconds and 60 seconds. So, the time after which all the three traffic signals will turn red from green together = LCM of 25, 39 and 60 seconds

$25 = 5^2$ $39 = 3 \times 13$ $60 = 2^2 \times 3 \times 5$

LCM of 25, 39 and 60 seconds = $5^2 \times 3 \times 2^2 \times 13$

= 3900 seconds = 65 minutes = 1 hour 5 minutes

Now, durations for green and red colours are same. So, the time after which all the three traffic signals will turn green from red together = 1 hour 5 minutes

So, the total time taken by all the three traffic signals to turn green together = 1 hour 5 minutes + 1 hour 5 minutes = 2 hours 10 minutes.

Thus, the time after which all the three traffic signals will simultaneously change to green again = 2:00 p.m. + 2 hours 10 minutes = 4:10 p.m.

Hence, option (b) is correct.

2. **What is the least four-digit number when divided by 3, 4, 5 and 6 leaves a remainder 2 in each case?**

   (*a*) 1012   (*b*) 1022

   (*c*) 1122   (*d*) 1222

Sol: We need to find the number that leaves remainder 2 when divided by 3, 4, 5 and 6.

So, required number – 2 will be divisible by 3, 4, 5 and 6.

So, required number – 2 = L.C.M of (3, 4, 5 and 6) = 60.

So, the required number be (60*x* + 2) where '*x*' is a natural number. i.e. (required number – 2) must be a multiple of 60.

For option (*a*): 1012 – 2 = 1010 is not a multiple of '60'

For option (*b*): 1022 – 2 = 1020 is a multiple of '60'.

Hence, option (b) is correct.

- If p and q are two numbers then,
  $(p \times q) = $ (HCF. of p and q) $\times$ (LCM of p and q)
- **L.C.M/H.C.F of Fractions:**
  - L.C.M $= \dfrac{L.C.M \text{ of numerators}}{H.C.F \text{ of denominators}}$
  - H.C.F $= \dfrac{H.C.F \text{ of numerators}}{L.C.M \text{ of denominators}}$

Lets understand this with the help of a questions:

1. **If the sum of two numbers is 40 and the HCF. and LCM of these numbers are 5 and 75 respectively. Find the sum of reciprocal of the two numbers.**

   (a) $\dfrac{1}{8}$　　　　　　(b) 8

   (c) $\dfrac{8}{75}$　　　　　　(d) $\dfrac{8}{15}$

Sol:

Let the two numbers be A and B

$A + B = 40$

$HCF = 5$ and $LCM = 75$

We know that,

Product of two numbers $=$ H.C.F $\times$ L.C.M

So, $A \times B = 5 \times 75$

And the sum of reciprocals $= \dfrac{1}{A} + \dfrac{1}{B} = \dfrac{A+B}{AB}$　　40　　8

$= \dfrac{40}{5 \times 75} = \dfrac{8}{75}$

Hence, option (c) is correct.

2. **What is the greatest length 'x' such that $3\dfrac{1}{2}\, m$ and $8\dfrac{3}{4}\, m$ are integral multiples of 'x'?**

   (a) $1\dfrac{1}{2}\, m$　　　　　　(b) $1\dfrac{1}{3}\, m$

   (c) $1\dfrac{1}{4}\, m$　　　　　　(d) $1\dfrac{3}{4}\, m$

Sol:

We know that x should be the H.C.F of $3\dfrac{1}{2}$ m and $8\dfrac{3}{4}$

m and, $3\dfrac{1}{2} = \dfrac{7}{2}$ and $8\dfrac{3}{4} = \dfrac{35}{4}$

'x' $=$ H.C.F $\left(\dfrac{7}{2}, \dfrac{35}{4}\right) = \dfrac{H.C.F \text{ of numerators}}{L.C.M \text{ of denominators}}$

$= \dfrac{H.C.F\ (7, 35)}{L.C.M\ (2, 4)} = \dfrac{7}{4} = 1\dfrac{3}{4}\, m$

Hence, option (d) is correct.

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3 Elementary Algebra

SOME IMPORTANT IDENTITIES

1. $(a + b)^2 = a^2 + 2ab + b^2$
2. $(a - b)^2 = a^2 - 2ab + b^2$
3. $a^2 - b^2 = (a + b)(a - b)$
4. $(a + b)^3 = a^3 + b^3 + 3ab(a + b)$
5. $(a - b)^3 = a^3 - b^3 - 3ab(a - b)$
6. $(a + b + c)^2 = a^2 + b^2 + c^2 + 2ab + 2bc + 2ca$
7. $(a + b + c)^3 = a^3 + b^3 + c^3 + 3(a + b)(b + c)(c + a)$

If $x + \dfrac{1}{x} = A$	$x^2 + \dfrac{1}{x^2} = A^2 - 2$	$x^3 + \dfrac{1}{x^3} = A^3 - 3A$
If $x - \dfrac{1}{x} = A$	$x^2 + \dfrac{1}{x^2} = A^2 + 2$	$x^3 - \dfrac{1}{x^3} = A^3 + 3A$

QUADRATIC EQUATIONS

1. Discriminant(D):

Consider a quadratic equation, $ax^2 + bx + c = 0$ then $b^2 - 4ac$ is known as a discriminant of the quadratic equation.

$$D = b^2 - 4ac$$

2. Nature of roots:

Nature of roots means whether the roots are real or non real, equal or unequal, zero or non – zero. It can be interpreted from the value of discriminant.

(i) If $D > 0$, then roots are real and unequal.

(ii) If $D = 0$, then the roots are real and equal. And each root is equal to $\left(-\dfrac{b}{2a} \right)$

(iii) If $D < 0$ then the roots are non real or imaginary. Imaginary roots are always in conjugate pair. It means if one root is ($a + ib$), then other root will be ($a – ib$) where $b \neq 0$.

3. Sum of roots and Product of roots:

Let α, β are real roots of the quadratic equation
$ax^2 + bx + c = 0$
Since, $ax^2 + bx + c = 0$
Dividing each term br 'a', we get;

$$x^2 + \left(\dfrac{b}{a} \right) x + \left(\dfrac{c}{a} \right) = 0 \qquad ...(i)$$

So, sum of roots $= \alpha + \beta = -\dfrac{b}{a}$

Ans, product of roots $= \alpha \times \beta = \dfrac{c}{a}$

SOLUTION OF TWO LINEAR EQUATIONS

To solve two linear equations, we need the equations in the standard form:

$$a_1 x + b_1 y = c_1 \qquad ...(i)$$
$$a_2 x + b_2 y = c_2 \qquad ...(ii)$$

Method 1: Substitution Method

From 1st equation find the value of y in term of x.

So from 1st equation $y = \dfrac{c_1 - a_1 x}{b_1}$

Now put the value of y (in terms of x) in 2nd equation.

So our 2nd equation will become linear equation with one variable, solve that to find the value of x and then put the value of x in equation $y = \dfrac{c_1 - a_1 x}{b_1}$ to find the value of y.

Method 2 : Elimination Method

Multiply one or both equations so that one variable has the same coefficient (or opposite coefficients).

Add or subtract the equations to eliminate one variable.

Solve for the remaining variable and substitute back to find the other.

Example: Solve the following linear equations:

$3x - 5y = - 30$ and $2x + 7y = 11$

Given equations are,

$3x - 5y = - 30 \qquad ...(i)$

$2x + 7y = 11 \qquad ...(ii)$

Multiply equation (1) by 2 and equation (2) by 3, we get

$6x - 10y = - 60 \qquad ...(iii)$

$6x + 21y = 33 \qquad ...(iv)$

Subtracting equation (iii) from (iv),

$31y = 93 \Rightarrow y = 3$

Putting the value of $y = 3$ in equation (1) and on solving we get $x = - 5$

Hence, the solution of the given equations is $(x, y) = (-5, 3)$

Let solve some questions on these concepts:

1. **The total cost of 4 oranges, 6 mangoes and 8 apples is equal to twice the total cost of 1 orange, 2 mangoes and 5 apples. Consider the following statements:**

   I. The total cost of 3 oranges, 5 mangoes and 9 apples is equal to the total cost of 4 oranges, 6 mangoes and 8 apples.

   II. The total cost of one orange and one mango is equal to the cost of one apple.

   Which of the statements given above is/are correct?

   (*a*) I only
   (*b*) II only
   (*c*) Both I and II
   (*d*) Neither I nor II

   Sol: Let the cost price of each orange, each mango and each apple be Rs. 'O', Rs. M' and Rs. 'A', respectively.

   So, $4O + 6M + 8A = 2(O + 2M + 5A)$
   Or, $4O + 6M + 8A = 2O + 4M + 10A$
   Or, $2O + 2M = 2A$
   Or, $A = O + M$

   For statement 'I':
   $3O + 5M + 9A = 4O + 6M + 8A$
   Or, $A = O + M$
   So, statement I is correct.

   For statement 'II':
   $O = M + A$
   So, statement II is correct.

   Hence, option (c) is correct.

2. **A person 'P' asks one of his three friends 'X' as to how much money he had. 'X' replied, "If 'Y' gives me Rs. 40, then 'Y' will have half of as much as 'Z', but if 'Z' gives me Rs. 40, then three of us will have equal amount." What is the total amount of money that 'X', 'Y' and 'Z' have?**

   (*a*) Rs. 420
   (*b*) Rs. 360
   (*c*) Rs. 300
   (*d*) Rs. 270

   Sol: Let the amount of money with 'X', 'Y' and 'Z' be Rs. 'x', Rs. 'y' and Rs. 'z', respectively.

   As per the question,

   (1) If 'Y' gives me (i.e. 'X') Rs. 40, then 'Y' will have half of as much as 'Z'.

   So, $y - 40 = \dfrac{z}{2}$

   Or, $z = 2y - 80$ ...(*i*)

   (2) If 'Z' gives me (i.e. 'X') Rs. 40, then three of us will have an equal amount.

   So, $x + 40 = y = z - 40$

   So, $x = y - 40$
   And, $z = y + 40$ ...(*ii*)

   From equations (*i*) and (*ii*), we have;
   $z = 2y - 80 = y + 40$
   Or, $2y - 80 = y + 40$
   Or, $y = 120$
   So, $x = y - 40 = 120 - 40 = 80$
   And $z = y + 40 = 120 + 40 = 160$

   So, money with 'X', 'Y' and 'Z' is Rs. 80, Rs. 120, and Rs. 160, respectively.

   Total amount of money with 'X', 'Y' and 'Z' $= x + y + z$
   $= 80 + 120 + 160 = $ Rs. 360

   Hence, option (b) is correct.

3. **Let p be a two-digit number and q be the number consisting of the same digits written in reverse order. If p × q = 2430, then what is the difference between p and q?**

   (*a*) 45
   (*b*) 27
   (*c*) 18
   (*d*) 9

   Sol: $p \times q = 2430$

   The last digit of the product is 0, which indicates that one of the two digits must be 5. The other possibility is one of the numbers ending with a 0, however 2-digit number multiplied with a single digit number shall not result in 2430 in any case (Highest possible case is $90 \times 09 = 810$

   Let the remaining digit be X.

   So, $p = $ '$X5$' $= 10X + 5$

   Reverse number, $q = $ '$5X$' $= 50 + X$

   According to the question,
   $(10X + 5) \times (50 + X) = 2430$
   $\Rightarrow 500X + 10X^2 + 250 + 5X = 2430$
   $\Rightarrow 10X^2 + 505X - 2180 = 0$
   $\Rightarrow 2X^2 + 101X - 436 = 0$
   $\Rightarrow 2X^2 + 109X - 8X - 436 = 0$
   $\Rightarrow X(2X + 109) - 4(2X + 109) = 0$
   $\Rightarrow (X - 4)(2X + 109) = 0$
   $\Rightarrow X = 4$, or $-\dfrac{109}{2}$ (neglected)
   So, $X = 4$

   Therefore,
   $p = 10 \times 4 + 5 = 45$
   $q = 50 + 4 = 54$
   Difference $= 54 - 45 = 9$

   Hence, option (d) is correct.

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An average or an arithmetic mean of a given set of observations is the sum of all the observations divided by the total number of observations. It is a very effective way of representing an entire group of data by a single value.

$$Average = \frac{\text{Sum of all observations}}{\text{Number of observations}}$$

Important Facts about Average:

1. If all the given quantities have the same value, then the average is equal to the number itself.

   Example: Let 10, 10, 10, 10 and 10 be the set of observations, then the average is 10.

2. If each quantity is increased/decreased by a value '*p*' then the new average is increased/decreased by *p*.

   Example: let the observations are:
   5, 7, 13, 12, 8, 15, 1, 9, 6, 4

Their average = $\dfrac{(5+7+13+12+8+15+1+9+6+4)}{10}$

$= \dfrac{80}{10} = 8$

Increasing each observation by 2:
5 + 2 = 7, 7 + 2 = 9, 13 + 2 = 15, 12 + 2 = 14,
8 + 2 = 10, 15 + 2 = 17, 1 + 2 = 3, 9 + 2 = 11,
6 + 2 = 8, 4 + 2 = 6.

So the average is $\dfrac{(7+9+15+14+10+17+3+11+8+6)}{10}$

= 10 (increased by 2)

3. Similarly if each quantity is multiplied/divided by a certain value p, then the new average is obtained by multiplying/dividing the old average by p.

TYPE OF QUESTIONS

Type 1: Average Age/Weight/Height based Questions

1. When a person leaves a group and another person joins the same group causing a change in the average age/weight/height of the group;

Case-I: Increase in average	Age/weight/height of new person = Age/weight/height of person who left the group + (Number of persons in the group including new person × Increase in average age/weight/height of the group)
Case-II: Decrease in average	Age/weight/height of new person = Age/weight/height of person who left the group – (Number of persons in the group including new person × Decrease in average age/weight/height of the group)

2. When a person leaves the group but nobody joins that group, then

Case I: Increase in average	Age/Weight/Height etc of person left = Previous average – (Number of persons present × Increase in average)
Case II: Decrease in average	Age/Weight/Height etc of person left = Previous average + (Number of persons present × Decrease in average)

Now lets understand these concepts by solving some questions :

1. The average age of 8 persons is increased by 2 years, when one of them, whose age is 24 years, is replaced by a new person. The age of the new person is :

   (*a*) 42 years

   (*b*) 40 years

   (*c*) 38 years

   (*d*) 45 years

Sol: According to the formula we read above

Age of new person can be determined as

= Age of person who left the group + (Number of persons in the group including new person × Increase in average age of the group)

Age of new person = 24 + (8 × 2) = 24 + 16 = 40

Age of new person = 40 years.

Hence, option (b) is correct.

2. **Consider the following:**

Weight of 6 boys = Weight of 7 girls = Weight of 3 men = Weight of 4 women.

If the average weight of the women is 63 kg, then what is the average weight of the boys?

(a) 40 kg   (b) 42 kg

(c) 45 kg   (d) 63 kg

Sol: Let average weight of a boy, a girl, a man and a woman is 'B' kg, 'G' kg, 'M' kg and 'W' kg, respectively.

According to question;

6B = 7G = 3M = 4W

So, 6B = 4W

Or, B = $\dfrac{4 \times 63}{6}$ = 42, as W = 63

Thus, average weight of the boys = 42 kg.

Hence, option (b) is correct.

3. **The average weight of 'A', 'B', 'C' is 40 kg, the average weight of 'B', 'D', 'E' is 42 kg and the weight of 'F' is equal to that of 'B'. What is the average weight of 'A', 'B', 'C', 'D', 'E' and 'F'?**

(a) 40.5 kg

(b) 40.8 kg

(c) 41 kg

(d) Cannot be determined as data is inadequate

Sol: Let weight of 'A', 'B', 'C', 'D', 'E' and 'F' be 'a' kg, 'b' kg, 'c' kg, 'd' kg, 'e' kg and 'f' kg, respectively.

Given average weight of 'A', 'B', 'C' = 40

$\dfrac{a+b+c}{3} = 40$

So, a + b + c = 120   …(i)

Average weight of 'B', 'D', 'E' = 42

$\dfrac{b+d+e}{3} = 42$,

So, b + d + e = 126   …(ii)

Given weight of 'F' = Weight of 'B'

So, $f = b$ …(iii)

Adding equations (i) and (ii), we have:

(a + b + c + b + d + e) = 120 + 126

Or, (a + b + c + d + e + b) = 246

Or, (a + b + c + d + e + f) = 246 [Since, b = f]

Thus, the average weight of 'A', 'B', 'C', 'D', 'E' and

'F' = $\dfrac{a+b+c+d+e+f}{6} = \dfrac{246}{6} = 41$

Hence, option (c) is correct.

Type 2: Average Speed Problems:

Average Speed = $\dfrac{\text{Total distance travelled}}{\text{Total time taken}}$

1. If a person covers distance 'A' km at 'a' km/hr and 'B' km at 'b' km/hr

Average speed of total journey = $\dfrac{A+B}{\left(\dfrac{A}{a}+\dfrac{B}{b}\right)} km/hr$

2. If a person travels from point 'P' to point 'Q' at a speed of 'a' km/hr and then returns back from point 'Q' to point 'P' at a speed of 'b' km/hr:

Average speed of total journey $= \dfrac{2ab}{a+b} km/hr$

3. If a person covers distance 'A' km at 'a' km/hr and 'B' km at 'b' km/hr and 'C' km at 'c' km/hr:

Average speed of total journey = $\dfrac{A+B+C}{\left(\dfrac{A}{a}+\dfrac{B}{b}+\dfrac{C}{c}\right)} km/hr$

4. If same distance is travelled with three different speeds 'a' km/hr, 'b' km/hr and 'c' km/hr, then:

Average speed of total journey = $\dfrac{3abc}{ab+bc+ca} km/hr$

5. If a person covers xth part of the distance at 'a traveled km/h, yth part of the distance at 'b traveled km/h and remaining zth part at 'c traveled km/h, then the average speed during entire journey

$= \left(\dfrac{1}{\dfrac{x}{a}+\dfrac{y}{b}+\dfrac{z}{c}}\right)$

Following Questions will help us in better understanding of these concepts:

1. **A car travels from a place X to place Y at an average speed of v km/hr, from Y to X at an average speed of $2v$ km/hr, again from X to Y at an average speed of $3v$ km/hr and again from Y to X at an average speed of $4v$ km/hr. Then, the average speed of the car for the entire journey**

(a) is less than v km/hr

(b) lies between v and $2v$ km/hr

(c) lies between $2v$ and $3v$ km/hr

(d) lies between $3v$ and $4v$ km/hr

Sol: Let the distance between X and Y be D.

Now, average speed = $\dfrac{\text{Total Distance covered}}{\text{Total time taken}}$

$= \dfrac{D+D+D+D}{\dfrac{D}{v}+\dfrac{D}{2v}+\dfrac{D}{3v}+\dfrac{D}{4v}}$

$$= \frac{4D}{\frac{D}{v}\left(1+\frac{1}{2}+\frac{1}{3}+\frac{1}{4}\right)} = \frac{4v}{\frac{50}{24}}$$

$$= \left(\frac{4 \times 24}{50}\right)v = 1.92v$$

Therefore, the average speed of the car for the entire journey lies between v and 2v km/hr.

Hence, option (b) is correct.

2. **Two cities A and B are 360 km apart. A car goes from A to B with a speed of 40 km/h and returns to A with a speed of 60 km/hr. What is the average speed of the car?** **(UPSC CSAT 2015)**

(*a*) 45 km/h   (*b*) 48 km/h

(*c*) 50 km/h   (*d*) 55 km/h

Sol: Given distance between the cities A and B = 360 km.

So, average speed $= \dfrac{\text{Total Distance covered}}{\text{Total time taken}}$

$$= \frac{360+360}{\frac{360}{40}+\frac{360}{60}} = \frac{720}{15} = 48 \, \text{km/h}$$

This is the the general solution by this we can find the solution of any question related to average speed.

We can solve this problem bu using the formula given above

Average speed $= \dfrac{2ab}{(a+b)}$

$$= \frac{2 \times 40 \times 60}{4+60} = \frac{4800}{100} = 48 \, \text{km/h}$$

Hence, option (b) is correct.

Type 3: Miscellaneous Questions from Average:

1. If the average of 'M' number of observations is 'N' but some observations are misread as 'a', 'b' and 'c' in place of 'x', 'y' and 'z' respectively, then the correct average

$$= \frac{M \times N - (a+b+c) + (x+y+z)}{M}$$

1. **The average marks of 100 students are given to be 40. It was found later that marks of one student were 53 which were misread as 83. The corrected mean marks are:** **(UPSC CSAT 2019)**

(*a*) 39   (*b*) 39.7

(*c*) 42   (*d*) 40

Sol: General **method:**

Given, average marks of 100 students = 40.

So, total marks of 100 students = 40 × 100 = 4000

But in this the total error is 83 – 53 = 30, more than the actual total.

So, Actual total = 4000 – 30 = 3970

So, the correct mean $= \dfrac{3970}{100} = 39.70$.

Alternate method: By formula

Here N = 40 and M = 100 and 53 was misread as 83.

So the correct mean $= \dfrac{40 \times 100 + 53 - 83}{100} = \dfrac{3970}{100}$

= 39.70

Hence, option (b) is correct.

2. **Weighted Average:**

Weighted average = (Summation of product of each value and its corresponding weights) ÷ (sum of weights)

$$= \frac{A_1 x_1 + A_2 x_2 + \dots + A_n x_n}{x_1 + x_2 + \dots + x_n}$$

Here, A_1, A_2, ..., A_n are the values while x_1, x_2, ..., x_n are the corresponding weights

2. **Consider the following data:**

	Average marks in English	Average marks in Hindi
Girls	9	8
Boys	8	7
Overall average marks	8.8	x

What is the value of 'x' in the above table? **(UPSC CSAT 2020)**

(*a*) 7.8   (*b*) 7.6

(*c*) 7.4   (*d*) 7.2

Sol: Let the number of girls be 'g' and the number of boys be 'b'.

Overall average marks of all the students in English

$$= \frac{\text{Total marks of all the students in English}}{\text{Total number of students}}$$

$$= \frac{9g + 8b}{g+b} = 8.8$$

$\Rightarrow 9g + 8b = 8.8g + 8.8b$

$\Rightarrow 0.2g = 0.8b$

$\Rightarrow g = 4b$ ...(i)

Similarly, overall average marks of all the students in

hindi $= \dfrac{\text{Total marks of all the students in Hindi}}{\text{Total number of students}}$

$$= \frac{8g + 7b}{g+b} = x$$

Putting value of 'g' from equation (i), we get:

$$= \frac{8 \times 4b + 7b}{4b+b} = \frac{39b}{5b} = 7.8$$

x = 7.8.

Hence, option (a) is correct.

KEY CONCEPTS

- **Percent = Per + cent (per hundred or out of hundred)**
- **Conversion of Fraction into Equivalent Percentage:**

  $\dfrac{a}{b}$ is a fraction which means "a" out of "b" to convert it in equivalent percentage we need to calculate the quantity

  out of hundred = $\dfrac{a}{b} \times 100$ (To Convert a fraction/Decimal into a percentage multiply it by 100.)

- **Conversion of Percentage into Fraction:** To convert a percent into a fraction divide it by 100.

  7.5% in fraction form $\Rightarrow$ 7.5% = $\dfrac{7.5}{100} = \dfrac{3}{40}$

- a% of b = $\dfrac{ab}{100}$ = b% of a

  For example: 40% of 120 = $\dfrac{120 \times 40}{100} = 48 = 120\%$ of 40

- **Fraction to Percentage Chart:**

Fraction	Percentage
1	100%
$\dfrac{1}{2}$	50%
$\dfrac{1}{3}$	33.33%
$\dfrac{1}{4}$	25%
$\dfrac{1}{5}$	20%
$\dfrac{1}{6}$	16.66%
$\dfrac{1}{7}$	14.28%
$\dfrac{1}{8}$	12.5%
$\dfrac{1}{9}$	11.11%
$\dfrac{1}{10}$	10%

TYPE OF QUESTIONS

Type 1: Percentage Increase/Decrease of a Quantity

1. If amount/quantity is increased/decreased, then

   % Increase/decrease = $\dfrac{\text{increase / decrease in value}}{\text{original value}} \times 100$

   E.g : If the fees for a mathematics course is increased from Rs. 5000 to Rs. 6500.

   % increase = $\dfrac{6500 - 5000}{5000} \times 100 = \dfrac{1500}{5000} \times 100 = 30\%$

2. If the quantity P increases or decreases at rate of $x\%$, $y\%$ and $z\%$ succesively then the final quantity is

   $= P\left(1 \pm \dfrac{x}{100}\right)\left(1 \pm \dfrac{y}{100}\right)\left(1 \pm \dfrac{z}{100}\right)$

   '+' sign indicates increase in quantity.

   '−' sign indicates decrease in quantity.

   Note: In these types of questions one can also assume a convenient known value for an unknown variable in the question to arrive at the answer. Most used value for this purpose is 100.

For better understanding lets solve some questions related to these concepts:

1. **If the price of an article is decreased by 20% and then the new price is increased by 25%, then what is the net change in the price?** **(UPSC CSAT 2021)**

   (*a*) 0%

   (*b*) 5% increase

   (*c*) 5% decrease

   (*d*) Cannot be determined due to insufficient data

Sol: Let the initial price is "*P*".

Now, the article is decreased by 20% and then increased by 25%.

here $x = -20$ and $y = 25$

According to the formula:

Final price = $P\left(1 - \dfrac{20}{100}\right) \times \left(1 + \dfrac{25}{100}\right) = P \times \dfrac{4}{5} \times \dfrac{5}{4} = P$

Hence the final price = P

So change = 0%

Alternate Solution:

Lets take the original price of the article = $100x$

Now the price is decreased by 20%

So, the new price = $100x - 20\%$ of $100x = 80x$

Now the price is increased by 25% then the final price = $80x + 25\%$ of $80x = 100x$

So, the change in original price and final price = $100x - 100x = 0$.

% change = 0%.

Hence, option (a) is correct.

2. **The increase in the price of a certain item was 25%. Then the price was decreased by 20% and then again increased by 10%. What is the resultant increase in the price?** **(UPSC CSAT 2022)**

(*a*) 5%   (*b*) 10%

(*c*) 12.5%   (*d*) 15%

Sol: Let the initial price is "P".

Now, the price is increased by 25 % then decreased by 20% and then increased by 10%.

Here $x = 25\%$, $y = -20\%$ and $z = 10\%$.

Now final price according to the formula

Final price = $P\left(1 + \dfrac{25}{100}\right) \times \left(1 - \dfrac{20}{100}\right)\left(1 + \dfrac{10}{100}\right)$

$= P \times \dfrac{5}{4} \times \dfrac{4}{5} \times \dfrac{11}{10} = \dfrac{11P}{10}$

Total change in price = $\dfrac{11P}{10} - P = \dfrac{P}{10}$

% Change = $\dfrac{\frac{P}{10}}{P} \times 100 = \dfrac{100}{10} = 10\%$

Alternate Solution:

Let the initial price of the item be Rs. '$100x$'.

After 25% increase in price, the new price of the item will be = $100x + 25\%$ of $100x =$ Rs. '$125x$'

Thereafter in case of 20% decrease in price, the new price of the item will be = $125x - 20\%$ of $125x =$ Rs. '$100x$'

Again after increase in price by 10%, the new price of the item will be = $100x + 10\%$ of $100x =$ Rs. '$110x$'

So, resultant percentage increase in price from initial

$= \left(\dfrac{110x - 100x}{100} \times 100\%\right) = 10\%$

Hence, option (b) is correct.

Type 2: Based on Income/Expenditure

1. If the cost of an article is increased by $x\%$, then how much to decrease the consumption of article, so that expenditure remains same is given by,

% decrease in consumption = $\left(\dfrac{x}{100 + x} \times 100\right)\%$

2. If the cost of an article is decreased by $x\%$, then how much to increase the consumption of article, so that expenditure remains same is given by,

% increase in consumption = $\left(\dfrac{x}{100 - x} \times 100\right)\%$

3. If the income of a person A is $x\%$ more than person B, then income of B is less in comparison to the A by,

$\left(\dfrac{x}{100 + x} \times 100\right)\%$

4. If the income of a person A is $x\%$ less than person B, then income of B is more in comparison to the A by,

$\left(\dfrac{x}{100 - x} \times 100\right)\%$

Following questions will help us in better understanding:

1. **If the price of an article is increased by 25%, then by what percent the consumption is to be decreased so that there is no change in expenditure?**

(*a*) 20%   (*b*) 25%

(*c*) 12.5%   (*d*) 10 %

Sol: According to formula:

% decrease in consumption = $\left(\dfrac{x}{100 + x} \times 100\right)\%$

$x = 25\%$

Hence, % decrease in consumption

$= \left(\dfrac{25}{100 + 25} \times 100\right)\%$

$= \left(\dfrac{25}{125} \times 100\right)\% = 20\%$

Alternate Solution:

Let us suppose that price of article is 100 for 100 units.

New price = $100 + 25\%$ of $100 = 125$.

So, the number of units the person can buy for Rs. 125 = 100 units

The number of units the person can buy for 1 rupee $= \dfrac{100}{125}$ units

The number of units the person can by for 100 Rs. $= \dfrac{100}{125} \times 100 = 80$ units.

Now the person can only buy 80 units.

Decrease in consumption = $100 - 80 = 20$ units (because the person could buy 100 units but after increase in price he can only buy 80 units)

% decrease in consumption = $\dfrac{20}{100} \times 100\% = 20\%$

Hence, option (a) is correct.

2. The full tank petrol in Arun's motorcycle lasts for 10 days. If he starts using 25% more everyday, how many days will the tank full petrol last?

(UPSC CSAT 2013)

(*a*) 5   (*b*) 6

(*c*) 7   (*d*) 8

Sol: Let us assume that Arun uses x units of petrol everyday. So, the amount of petrol in the tank when it is full will be 10x. If he starts using 25% more petrol everyday then the units of petrol he now used everyday will be

$$x\left(1+\frac{25}{100}\right)=1.25x$$

So, the number of days his petrol will now last

$$=\frac{\text{Amount of petrol in tank}}{\text{Number of units used everyday}}=\frac{10x}{1.25x}=8$$

Hence, option (d) is correct.

Type 3: Based on Poulation/Price/Quantity

If the present population of a town is P and the population increases or decreases at rate of *x*%, *y*% and *z*% in the first, second and third year respectively.

○ Then the population of town after 3 years

$$= P\left(1\pm\frac{x}{100}\right)\left(1\pm\frac{y}{100}\right)\left(1\pm\frac{z}{100}\right)$$

'+' sign indicates increase in population.

'–' sign indicates decrease in population.

Following question will help us in better understanding:

1. The population of a town decreases at the rate of 10% for the first year, it increases at the rate of 22% for the second year and for the third year it is increased at the rate of 15%. Then what will be the population after 3 years if the present population of the town is 3250000?

(*a*) 4107375   (*b*) 4103775

(*c*) 4105773   (*d*) 4107735

Sol: Using formual

Population after 3 years

$$= P\left(1-\frac{x}{100}\right)\left(1+\frac{y}{100}\right)\left(1+\frac{z}{100}\right)$$

Given, P = 3250000,

x = – 10%, *y* = 22%, *z* = 15%

Population of twon after 3 years

$$= P\left(1-\frac{10}{100}\right)\left(1+\frac{22}{100}\right)\left(1+\frac{15}{100}\right)$$

$$= 3250000\left(\frac{90}{100}\right)\left(\frac{122}{100}\right)\left(\frac{115}{100}\right)$$

= 3250000(0.9) (1.22) (1.15)

= 4103775

Hence, option (b) is correct.

Type 4: Based on Examination Marks

1. The percentage of passing marks in an examination is *a*%. If *a* candidate who scores *b* marks fails by *c* marks, then the maximum marks (*M*) are given by,

$$M = \frac{100(b+c)}{a}$$

2. An examinee scored *x*% marks in an exam, and failed by '*a*' mark. In the same examination another examinee obtained *y*% marks and passed with '*b*' more marks than minimum, then, Maximum marks = $\dfrac{100}{(y-x)}\times(a+b)$

3. In a certain examination, '*x*' boys and 'y girls participated. *a*% of boys and *b*% of girls passed the examination, then, Percentage of passed students of the total students =

$$\left(\frac{x\times a+y\times b}{x+y}\right)\%$$

Following question will help us in better understanding:

1. A student has to get 40% marks to pass in an examination. Suppose he gets 30 marks and fails by 30 marks, then what are the maximum marks in the examination? **(UPSC CSAT 2018)**

(*a*) 100

(*b*) 120

(*c*) 150

(*d*) 300

Sol: Method **1: By formula:**

40% marks is required to pass an examination. The percentage of passing marks in an examination is a %. If a candidate who scores b marks fails by c marks, then the maximum marks(M)

$$M = \frac{100(b+c)}{a} \quad \text{here } a = 40\ \%\ ,\ b = 30,\ c = 30$$

$$M = \frac{100(30+30)}{40} = 150$$

Method 2: General method:

Let x be the total marks.

So, 40% of *x* = passing marks.

Given, the student got 30 marks and failed by 30 marks.

Therefore passing mark = 60 marks

So, 40% of *x* = 60 or $\dfrac{40x}{100}$ = 60 or *x* = 150 marks.

Hence, option (c) is correct.

2. In an examination, 'A' has scored 20 marks more than 'B'. If 'B' has scored 5% less marks than 'A', how many marks has 'B' scored?

(UPSC CSAT 2019)

(*a*) 360   (*b*) 380

(*c*) 400   (*d*) 420

Sol: Let marks scored by '*A*' be '*x*'

Then, marks scored by '*B*' = 0.95 × *x* = '0.95*x*'

According to the question;

$x - 0.95x = 20$

Or, $0.05x = 20$

Or, $x = 400$

Marks scored by '*B*' = 0.95 × 400 = 380

Hence, option (b) is correct.

3. Candidates in a competitive examination consisted of 60% men and 40% women. 70% men and 75% women cleared the qualifying test and entered the final test where 80% men and 70% women were successful. Which of the following statements is correct? **(UPSC CSAT 2015)**

(*a*) Success rate is higher for women.

(*b*) Overall success rate is below 50%.

(*c*) More men cleared the examination than women.

(*d*) Both (*a*) and (*b*) above are correct.

Sol: Let the total number of candidates in the beginning be '1000*x*'

Number of men = 60% of 1000*x* = '600*x*'

Number of women = 40% of 1000*x* = '400*x*'

Number of men who cleared the qualifying test = 70% of 600*x* = '420*x*'

Number of women who cleared the qualifying test = 75% of 400*x* = '300x'

Number of men who succeeded in the final test = 80% of 420*x* = 0.8 × 420*x* = '336*x*'

Number of women who succeeded in the final test = 70% of 300*x* = 0.7 × 300*x* = '210*x*'

Success rate for men = $\left(\dfrac{336x}{600x} \right)$ × 100 = 56%

Success rate for women = $\left(\dfrac{210x}{400x} \right)$ × 100 = 52.5%

Overall success rate = $\left[\dfrac{(336x + 210x)}{1000x} \right]$ × 100 = 54.6%

Hence, option (c) is correct.

4. A student appeared in 6 papers. The maximum marks are the same for each paper. His marks in these papers are in the proportion 5:6:7:8:9:10. Overall he scored 60%. In how many number of papers did he score less than 60% of the maximum marks? **(UPSC CSAT 2021)**

(*a*) 2   (*b*) 3

(*c*) 4   (*d*) 5

Sol: In this type of question take the total marks of each paper = 100*a* (*w*ill be easier to calculate percentage)

Therefore, total marks for all 6 papers

= 6 × 100*a* = '600*a*'

Overall marks scored = 60% of 600*a* = '360*a*'

Let the marks obtained in the 6 papers are:

'5*x*', '6*x*', '7*x*', '8*x*', '9*x*' and '10*x*'

Thus, $5x + 6x + 7x + 8x + 9x + 10x = 360a$

Or, $45x = 360a$ or, $x = 8a$

So, marks in the given 6 papers must be:

Marks obtained in paper 1: 5 × 8*a* = 40*a*

Percentage of marks obtained = $\dfrac{40a}{100a} \times 100$ = 40% < 60%

Marks obtained in paper 2: 6 × 8*a* = 48*a*

Percentage of marks obtained = $\dfrac{48a}{100a} \times 100$ = 48% < 60%

Marks obtained in paper 3: 7 × 8*a* = 56*a*

Percentage of marks obtained = $\dfrac{56a}{100a} \times 100$ = 56% < 60%

Marks obtained in paper 4: 8 × 8*a* = 64*a*

Percentage of marks obtained = $\dfrac{64a}{100a} \times 100$ = 64% > 60%

Marks obtained in paper 5: 9 × 8*a* = 72*a*

Percentage of marks obtained = $\dfrac{72a}{100a} \times 100$ = 72% > 60%

Marks obtained in paper 6: 10 × 8*a* = 80*a*

Percentage of marks obtained = $\dfrac{80a}{100a} \times 100$ = 80% > 60%

Hence, in 3 papers the student has scored less than 60% marks.

Hence, option (b) is correct.

❖   ❖   ❖

Ratio, Mixture, Partnership and Proportion

RATIO

The ratio is a way of comparing two quantities in terms of magnitude. It tells us how many times one quantity is greater or lesser than the other.

For example: A has 3 books and B has 5 books. It implies that the ratio of number of books between A and B is 3 to 5. Which is expressed as 3:5.

So the ratio of any two quantities is expressed as a/b or a:b.

Properties and Important Results of Ratio

1. Value of the ratio does not change when the numerator and denominator both are multiplied or divided by the same number or quantity.

2. If the two numbers $a$re in the ratio $a : b$ and the sum of these numbers is A, then these number will be $\dfrac{aA}{a+b}$ and $\dfrac{bA}{a+b}$.

Now lets solve some questions based on these concepts to get a better understanding:

1. **The ratio of a two-digit natural number to a number formed by reversing its digits is 4 : 7. The number of such pairs is** **(UPSC CSAT 2019)**

   (*a*) 5   (*b*) 4
   (*c*) 3   (*d*) 2

Sol: As we learned in the number system any two-digit number "ab" can be written as $10a + b$ and after reversing the digits the number will be $10b + a$.

Now according to the question: "$10a + b$" : "$10b + a$" = 4:7

So we can write $\dfrac{10a+b}{10b+a} = \dfrac{4}{7}$; which gives $b = 2a$.

Therefore, in total, 4 pairs are possible (12, 21) (24, 42) (36, 63) (48, 84).

   Hence, option (b) is correct.

2. **The monthly incomes of X and Y are the ratio of 4 : 3 and their monthly expenses are in the ratio of 3 : 2. However, each saves Rs. 6000 per month. What is their total monthly income?** **(UPSC CSAT 2017)**

   (*a*) Rs. 28000   (*b*) Rs. 42000
   (*c*) Rs. 56000   (*d*) Rs. 84000

Sol: It is given the ratio of monthly incomes of X and Y is 4:3.

So we can assume the income of X = $4x$ and income of Y = $3x$

Similarly, their expenses are $3y$ and $2y$ respectively,

Saving = Income – Expenditure

Then according to question, $4x – 3y = 6000$ and $3x – 2y = 6000$

By solving linear equation we get $x = y = 6000$

So, income of X = $4x$ = 24000

And, Income of Y = $3x$ = 18000

Their total income = 24000 + 18000 = 42000

Hence, option (b) is correct.

3. **A sum of Rs. 2,500 is distributed among 'X', 'Y' and 'Z' in the ratio $\dfrac{1}{2} : \dfrac{3}{4} : \dfrac{5}{6}$. What is the difference between the maximum share and the minimum share?**

   (*a*) Rs. 300   (*b*) Rs. 350
   (*c*) Rs. 400   (*d*) Rs. 450

Sol:

Given ratio : $\dfrac{1}{2} : \dfrac{3}{4} : \dfrac{5}{6}$

LCM of denominators of the fractions = 12 [LCM of (2, 4 and 6)]

Now according to the property of the ratio "Value of the ratio does not change when the numerator and denominator both are multiplied or divided by the same number or quantity."

So multiply the ratio by 12 we get the ratio as: 6 : 9 : 10.

Let the share of 'X', 'Y' and 'Z' be Rs. '6n', Rs. '9n' and Rs. '10n', respectively.

Now, according to the question,

$6n + 9n + 10n = 2500$

Or, $25n = 2500$

Or, $n = 100$

Now, the difference between maximum share and minimum share = $10n – 6n$ = Rs. $4n$

= 4×100 = Rs. 400

By using formula:

We know that If the two numbers are in the ratio $a : b$ and the sum of these numbers is A, then these number will be $\dfrac{aA}{a+b}$ and $\dfrac{bA}{a+b}$.

Now we have 3 numbers x, y, and z are in the ratio $6 : 9 : 10$.

And the sum of these 3 numbers = 2500.

So the numbers will be x = $\dfrac{6 \times 2500}{6+9+10} = \dfrac{15000}{25} = 600$

$y = \dfrac{9 \times 2500}{6+9+10} = \dfrac{22500}{25} = 900$

$z = \dfrac{10 \times 2500}{6+9+10} = \dfrac{25000}{25} = 1000$

Hence, the difference between maximum and minimum = 1000 − 600 = 400.

Hence, option (c) is correct.

4. **The monthly incomes of Peter and Paul are in the ratio of 4:3. Their expenses are in the ratio of 3:2. If each saves Rs. 6,000 at the end of the month, their monthly incomes respectively are (in Rs.)**

(UPSC CSAT 2015)

(a) 24,000 and 18,000   (b) 28,000 and 21,000

(c) 32,000 and 24,000   (d) 34,000 and 26,000

Ans: Let the income of Peter and Paul be Rs. '$4x$' and Rs. '$3x$', respectively, and their expenses be Rs. '$3y$' and Rs. '$2y$', respectively

Since, Income – Expenses = Savings

According to question

Peter's savings = 4x − 3y = 6000 ...(i)

Paul's savings = 3x − 2y = 6000 ...(ii)

Equating equation (i) and (ii), we have;

$4x − 3y = 3x − 2y$

Or, $x = y$

Putting this value in equation (i), we get;

Peter's savings = $4x − 3y = 4x − 3x = x = 6000$

So, Paul's savings = $3x − 2y = 3x − 2x = x = 6000$

So, monthly income of Peter = $4x = 4 \times 6000$ = Rs. 24,000

And, monthly income of Paul = $3x = 3 \times 6000$ = Rs. 18,000

Monthly income of Peter and Paul be Rs. 24,000 and Rs. 18,000, respectively.

Hence, option (a) is correct.

PROPORTION

An equality of two ratios is called a proportion and we say that four numbers are in proportion. i.e if $a : b = c : d$ or $\dfrac{a}{b} = \dfrac{c}{d}$ then we can say that a, b, c and d are in proportion which can be written as : $a : b :: c : d$, where "::" is symbol for proportion as can be read as "a is to b as c is to d."

TEST OF PROPORTIONALITY:

If we have four numbers in proportion i.e., $a : b :: c : d$ then $a \times d = b \times c$

Now let's understand this concept with the help of some questions:

1. **A lift has the capacity of 18 adults or 30 children. How many children can board the lift with 12 adults?**

(UPSC CSAT 2018)

(a) 6   (b) 10

(c) 12   (d) 15

Sol: Let the weight of each adult and each child be 'x' kg and 'y' kg, respectively.

Since, the lift has a capacity of 18 adults or 30 children.

So, weight of 18 adults = weight of 30 children

Or, $18 \times x = 30 \times y$ or $x : y = 5 : 3$

Now, let $x = 5k$ and $y = 3k$

So, capacity of lift = $18x = 18 \times 5k$ = '$90k$' kg

Weight of 12 adults = $12 \times x = 12x = 12 \times 5k$ = '$60k$' kg

So, the extra weight that can be accommodated in the lift = $90k − 60k$ = '$30k$' kg

Thus, number of children that can board

$= \dfrac{30k}{\text{weight of each child}} = \dfrac{30k}{y} = \dfrac{30k}{3k} = 10$

Hence, option (b) is correct.

2. **A student appeared in 6 papers. The maximum marks are the same for each paper. His marks in these papers are in the proportion of 5:6:7:8:9:10. Overall he scored 60%. In how many papers did he score less than 60% of the maximum marks?**

(UPSC CSAT 2021)

(a) 2   (b) 3

(c) 4   (d) 5

Sol: Let the maximum marks in each subject = 100

Total maximum marks in 6 subjects = 100 + 100 + 100 + 100 + 100 + 100 = 600

Marks scored = 60% of 600 = 360

His marks in these papers are in the proportion of 5:6:7:8:9:10.

Let the marks in each paper be $5x$, $6x$, $7x$, $8x$, $9x$ and $10x$

$5x + 6x + 7x + 8x + 9x + 10x = 360$

$45x = 360$ or $x = 8$

Marks in each of the 6 subjects are $5 × 8 = 40$, $6 × 8 = 48$, $7 × 8 = 56$, $8 × 8 = 64$, $9 × 8 = 72$, $10 × 8 = 80$

So, in 3 subjects he scored less than 60% marks (40, 48 and 56).

Hence, option (b) is correct.

PARTNERSHIP

When a group of people come together to start a business by contributing their money and resources, we call it a joint venture or a partnership.

Basic Formula for Partnership:

Ratio of profit of two partners $P_1 : P_2$ is equal to ratio of product of their investment (say, I_1 and I_2) and time period of investment (say, T_1 and T_2) i.e $P_1 : P_2 = (I_1 × T_1) : (I_2 × T_2)$

For example: If A and B invested in ratio of 5 : 4 for 1 yr and 2 yr respectively then ratio of their profit $P_1 : P_2 = 5 × 1 : 4 × 2 = 5 : 8$

Let us understand this concept with the help of the following example:

1. **Arun starts some business with Rs. 70000. After 4 months Akash joins him with Rs. 80000. At the end of the year, in what ratio should they share the profits?**

   (*a*) 21:16   (*b*) 21:23

   (*c*) 17:27   (*d*) 13:24

Sol: Ratio of their investment = 70000 : 80000

Ratio of time period = 12 : 8

∴ Ratio of their shares in total profit = 70000 × 12 : 80000 × 8 = 21 : 16

Hence, option (a) is correct.

2. **Two persons P and Q enter into a business. P puts ₹14,000 more than Q, but P has invested for 8 months and Q has invested for 10 months. If P's share is ₹400 more than Q's share out of the total profit of ₹ 2,000, what is the capital contributed by P?**

   (UPSC CSAT 2024)

   (*a*) ₹ 30,000   (*b*) ₹ 26,000

   (*c*) ₹ 24,000   (*d*) ₹ 20,000

Ans: Let 'Q' contribute Rs. 'x'

So, 'P' contribute = Rs. (x + 14000)

Let profit share of 'Q' is Rs. 'y'

So, profit share of 'P' = Rs. (y + 400)

Thus, y + y + 400 = 2000

Or, 2y = 1600

So, y = 800

Therefore, profit share of 'Q' = Rs. 800

And, profit share of 'P = 400 + 800 = 1200

According to question;

$$\frac{(x + 14000) × 8}{x × 10} = \frac{1200}{800}$$

Or, 8x + 112000 = 15x

Or, 7x = 112000

Or, x = 16000

So, 'P' contribute = 16000 + 14000

= Rs. 30,000

Hence, option (a) is correct.

MIXTURE

1. When you combine two or more things in a specific amount, the result is a new product. We call this new product a mixture of the ingredients you used.

Alligation:

Alligation is a rule used to solve problems involving the mixing of two or more quantities with different prices, concentrations, or values. Here's a step-by-step guide to solve alligation-based questions:

If we are mixing two types of items such that one is priced at Rs. H per/kg and other is priced at Rs. L per/kg and the quantities of the given items mixed are 'A' kg and 'B' kg respectively then the mean price (M) such that $H > M > L$ of the mixture can be calculated as follows:

$$\frac{M - L}{H - M} = \frac{A}{B}$$
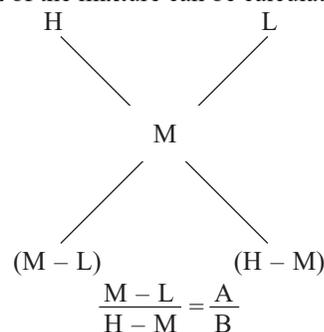
Following Questions will help us in better understanding:

1. **In what ratio must the grocer mix two types of wheat costing Rs. 6.50 per kg and Rs. 10 per kg respectively, so as to get a mixture worth Rs. 7.25 per kg?**

   (*a*) 11 : 3   (*b*) 3 : 11

   (*c*) 4 : 5   (*d*) 5 : 4

Sol: Here, L = Rs. 6. 50 per kg

H = Rs. 10 per kg

And, M = Rs. 7.25 per kg

So according to the formula the ratio is:
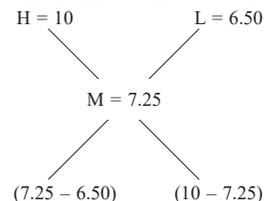
Required ratio = (10 – 7.25) : (7.25 – 6.50)

= 2.75 : 0.75 = 11 : 3

Hence, option (a) is correct.

> **2.** If there's a container with x units of pure liquid and you take out y units, replacing it with water. After doing this n times, the amount of pure liquid left in the container can be found using this given formula:
>
> Quantity of pure liquid left = $\left[x\left(1-\dfrac{y}{x}\right)^n \right]$

Let's understand the given concept with the help of a question:

2. A bottle contains 20 litres of liquid 'A'. 4 litres of liquid 'A' is taken out of it and replaced by same quantity of liquid 'B'. Again 4 litres of the mixture is taken out and replaced by same quantity of liquid 'B'. What is the ratio of quantity of liquid 'A' 'to that of liquid 'B' in the final mixture? (UPSC CSAT 2020)

(a) 4:1   (b) 5:1

(c) 16:9   (d) 17:8

Sol: **Using the formula:** If there's a container with x units of liquid and you take out y units, replacing it with water. After doing this n times, the amount of pure liquid left in the container can be found using this given formula:

Quantity of pure liquid left = $\left[x\left(1-\dfrac{y}{x}\right)^n \right]$

Here $x = 20l$, $y = 4l$ and $n = 2$

So, the quantity of liquid 'A' in the final mixture

$= 20 \times \left(1-\dfrac{4}{20}\right)^2 = 12.8l$

So, the quantity of liquid 'B' in the final mixture = 20 – 12.8 = 7.2 l

Required ratio A : B in final mixture = 12.8 : 7.2 = 16 : 9

Hence, option (c) is correct.

3. There are two containers 'X' and 'Y'. 'X' contains 100 ml of milk and 'Y' contains 100 ml of water. 20 ml of milk from 'X' is transferred to 'Y'. After mixing well, 20 ml of the mixture in 'Y' is transferred back to 'X'. If 'm' denotes the proportion of milk in 'X' and 'n' denotes the proportion of water in 'Y', then which one of the following is correct?

(UPSC CSAT 2022)

(a) $m = n$

(b) $m > n$

(c) $m < n$

(d) Cannot be determined due to insufficient data

Sol: It is given that container 'X' contains 100 ml of milk and container 'Y' contains 100 ml of water.

Quantity of milk left in container 'X' after transferring 20 ml of milk from container 'X' to 'Y'

= 100 – 20 = 80 ml

Quantity of mixtue in container 'Y' = 100 ml of water + 20 ml of milk = 120 ml

The ratio of the quantity of milk and water in container 'Y' = 20 : 100 = 1:5

Quantity of milk in 20 ml solution of container Y

$= \left(\dfrac{1}{6}\right) \times 20 = \left(\dfrac{10}{3}\right)$ ml

Quantity of water in 20 ml solution

$= 20 - \left(\dfrac{10}{3}\right) = \left(\dfrac{50}{3}\right)$ ml

Quantity of milk in container 'Y'

$= 20 - \left(\dfrac{10}{3}\right) = \left(\dfrac{50}{3}\right)$ ml

Quantity of water in container 'Y

$= 100 - \left(\dfrac{50}{3}\right) = \left(\dfrac{250}{3}\right)$ ml

'n' = Proportion of water in 'Y'

$= \dfrac{\text{Quantity of water in} 'Y'}{\text{Total Quantity of mixture in} 'Y'}$

$= \dfrac{\dfrac{250}{3}}{\dfrac{250}{3}+\dfrac{50}{3}} = \dfrac{250}{300} = \dfrac{5}{6}$

Quantity of milk in container 'X' = $80 + \left(\dfrac{10}{3}\right)$

$= \left(\dfrac{250}{3}\right)$ ml

Quantity of water in container

$'X' = \left(\dfrac{50}{3}\right)$ ml

'm' = proportional of milk in 'X'

$= \dfrac{\text{Quantity of water in} 'Y'}{\text{Total Quantity of mixture in} 'Y'}$

$= \dfrac{\dfrac{250}{3}}{\dfrac{250}{3}+\dfrac{50}{3}} = \dfrac{250}{300} = \dfrac{5}{6}$

Thus, $m = n$

Hence, option (a) is correct.

❖   ❖   ❖

Profit, Loss and Discount

TERMINOLOGY

Cost Price (CP): The amount paid by the shopkeeper to the manufacturer or wholesaler for the purchase of goods is referred to as the cost price (CP) of the acquired goods.

Selling Price (SP): The selling price (SP) is the amount at which the shopkeeper sells the goods to customers.

Profit: Profit occurs when the selling price of an article is higher than its cost price.

In terms of calculation, Profit = SP – CP, where SP is greater than CP.

Loss: Loss is incurred when the selling price of an article is less than its cost price.

The loss is calculated as Loss = CP − SP, where CP is greater than SP.

Basic Formulas Related to Profit and Loss

(i) Profit percentage = $\dfrac{profit}{cost\ price} \times 100 = \left(\dfrac{SP-CP}{CP}\right) \times 100$

(ii) Loss percentage = $\dfrac{loss}{cost\ price} \times 100 = \left(\dfrac{CP-SP}{CP}\right) \times 100$

(iii) SP = $\left(\dfrac{100+\text{gain}\%}{100} \times CP\right) = \left(\dfrac{100-\text{loss}\%}{100} \times CP\right)$

(iv) CP = $\left(\dfrac{100}{100+\text{gain}\%} \times SP\right) = \left(\dfrac{100}{100-\text{loss}\%} \times SP\right)$

(v) SP = (100 + k)% of CP; when profit = k% of CP

(vi) SP = (100 – k)% of CP; when profit = k% of CP

Now lets understand the method to solve the questions using these formulae:

1. **A person bought a car and sold it for Rs. 3,00,000. If he incurred a loss of 20%, then how much did he spend to buy the car?** **(UPSC CSAT 2020)**

    (*a*) Rs. 3,60,000   (*b*) Rs. 3,65,000

    (*c*) Rs. 3,70,000   (*d*) Rs. 3,75,000

Sol: **By Formula:**

In this question we are given: Selling price (SP) = Rs. 3,00,000, Loss% = 20% and we need to find out the cost price (CP).

So according to the formula:

CP = $\left(\dfrac{100}{100-loss}\right) \times SP = \left(\dfrac{100}{100-20}\right) \times 3,00,000$

= $\dfrac{100}{80} \times 3,00,000$ = Rs. 3,75,000

Another Method:

Suppose the cost price = Rs. 100

Given that loss = 20%

So the selling price = 100 – 20% of 100 = Rs. 80

So, if SP is Rs. 80 then CP = Rs. 100

If SP is Re. 1 then CP = Rs. $\dfrac{100}{80}$

And if SP is Rs. 3,00,000 then CP = $\dfrac{100}{80} \times 3,00,000 =$ Rs. 3,75,000.

Hence, option (d) is correct.

2. **A shopkeeper sells an article at Rs. 40 and gets X % profit. However, when he sells it at Rs. 20, he faces the same percentage of loss. What is the original cost of the article ?** **(UPSC CSAT 2018)**

    (*a*) Rs. 10   (*b*) Rs. 20

    (*c*) Rs. 30   (*d*) Rs. 40

Sol: Let the cost price of article be '*a*'

Formula: Cost Price $\left(1+\dfrac{\text{Gain/Profit \%}}{100}\right)$

= Selling Price & Cost Price $\left(1-\dfrac{\text{Loss \%}}{100}\right)$ = Selling Price

So, $a \times \left(1+\dfrac{X}{100}\right) = 40$

Or, $100a + aX = 4000$ ...(*i*)

Also, $a \times \left(1-\dfrac{X}{100}\right) = 20$

Or, $100a - aX = 2000$ ...(*ii*)

Equating equation (*i*) and (*ii*), we have

$aX = 100a - 2000 = 4000 - 100a$

Or, $200a = 6000$

Or, $a = 30$

Thus, the cost price of article is Rs. 30

Hence, option (c) is correct.

3. **If Sohan, while selling two goats at the same price, makes a profit of 10% on one goat and suffers a loss of 10% on the other.** **(UPSC CSAT 2014)**

   (a) he makes no profit and no loss.

   (b) he makes a profit of 1%

   (c) he suffers a loss of 1%

   (d) he suffers a loss of 2%

Sol: **General Method:**

The selling price of two goats are equal

Let the selling price of each goat be Rs. '99x'

Total selling price of both goats = 2 × 99x = Rs. '198x'

Cost price of the goat sold at a profit of 10% = $\left(\dfrac{99x}{1.10}\right)$ = Rs. '90x'

Cost price of the goat sold at a loss of 10% = $\left(\dfrac{99x}{0.9}\right)$ = Rs. '110x'

So, total selling price = 110x + 90x = Rs. '200x'

Loss = 200x – 198x = Rs. '2x'

Required Loss% = $\dfrac{2x}{200x} \times 100$ = 1%

By Formula:

If a shopkeeper sells two item at same price and makes a profit of R% on one item and loss of R% on other item then he suffers an overall loss of $\dfrac{R^2}{100}\%$

Using this formula in our question:

We have R = 10

So overall loss = $\dfrac{10^2}{100} = 1\%$

Hence, option (c) is correct.

DISCOUNT

A discount is a decrease in the original price of an item, which is known as the marked price. Discounts are always calculated on the marked price of the item.

Marked Price: The amount mentioned on the tag or label of a product is called the marked price or list price.

Important Formulae

(i) Selling price = Marked price – Discount

(ii) Discount = Marked price – Selling price

(iii) Discount % = $\left(\dfrac{\text{Discount}}{\text{Marked price}}\right) \times 100$

(iv) If discount allowed is r% then selling price

$= \left(\dfrac{100-r}{100} \times \text{marked price}\right)$

SUCCESSIVE DISCOUNTS FOR THREE TERMS

Suppose there are three successive discounts on an item, r_1%, r_2% and r_3%. The selling price of the item after r_1%, r_2% and r_3% discount.

$= \left(\dfrac{100-r_3}{100}\right) \times \left(\dfrac{100-r_2}{100}\right)\left(\dfrac{100-r_1}{100}\right) \times P$

$= \left(1-\dfrac{r_3}{100}\right) \times \left(1-\dfrac{r_2}{100}\right) \times \left(1-\dfrac{r_1}{100}\right)P$

Now lets understand these concepts through questions:

1. **Rakesh had money to buy 8 mobile handsets of a specific company. But the retailer offered a very good discount on that particular handset. Rakesh could buy 10 mobile handsets with the amount he had. What was the discount the retailer offered?**

   (UPSC CSAT 2019)

   (a) 15%   (b) 20%

   (c) 25%   (d) 30%

Sol: Let's assume the price of each mobile as Rs. '100x'

Total amount needed to buy 8 mobiles

= 8 × 100x = Rs. '800x'

Price of 10 mobiles = 10 × 100x = Rs. '1000x'

As, Rakesh got 10 mobiles in the price of 8 mobiles, thus required discount = 1000x – 800x = Rs. '200x'

So, the discount percentage offered

$= \left(\dfrac{200x}{1000x}\right) \times 100 = 20\%$

Alternate solution:

Discount (D) = Marked price (MP) – Selling price (SP)

Let us suppose Rakesh had ₹ 100

Price of each phone before discount = $\dfrac{100}{8}$ = ₹ 12.5

Price of each phone before discount = $\dfrac{100}{10}$ = ₹ 10

Discount% = $\left(\dfrac{\text{Discount}}{\text{Marked price}}\right) \times 100$

$\Rightarrow \left(\dfrac{12.5-10}{12.5}\right) \times 100$

$\Rightarrow \left(\dfrac{2.5}{12.5}\right) \times 100 \Rightarrow 0.2 \times 100$

$\Rightarrow 20\%$

Hence, option (b) is correct.

2. **A person allows a 10 % discount for cash payment from the marked price of a toy and still he makes a 10 % gain. What is the cost price of the toy which is marked Rs. 770?** **(UPSC CSAT 2016)**

   (a) Rs. 610   (b) Rs. 620

   (c) Rs. 630   (d) Rs. 640

Sol: MP = ₹ 770

CP = x

Discount = 10% of MP = $\dfrac{10}{100} \times 770$ = ₹ 77

Price after Discount = 770 – 77 = 693 = SP

$\Rightarrow$ Gain = 693 – $x = \dfrac{10}{100} \times x$

$\Rightarrow$ 693 – x = 0.1x

$\Rightarrow$ 1.1x = 693

$\Rightarrow x$ = 630

$\Rightarrow$ CP = 630

Hence, option (c) is correct.

3. **A shop owner offers the following discount options on an article to a customer:**

   I. Successive discounts of 10% and 20%, and then pay a service tax of 10%.

   II. Successive discounts of 20% and 10%, and then pay a service tax of 10%.

   III. Pay a service tax of 10% first, then successive discounts of 20% and 10%.

   Which one of the following is correct?

   (UPSC CSAT 2020)

   (*a*) I only is the best option for the customer.

   (*b*) II only is the best option for the customer.

   (*c*) III only is the best option for the customer.

   (*d*) All the options are equally good for the customer.

Sol: Let the base price be Rs. '100x'

Condition I:

Price after first discount = 100x – 10% × 100x = Rs. '90x'

Price after second discount = 90x – 20% × 90x

= Rs. '72x'

Price after Service tax = 72x + 10% × 72x = Rs. '79.2x'

Condition II:

Price after first discount = 100x – 20% × 100x = Rs. '80x'

Price after second discount = 80x – 10% × 80x

= Rs. '72x'

Price after Service tax = 72x + 10% × 72x = Rs. '79.2x'

Condition III:

Price after Service tax = 100x + 10% × 100x = Rs. '110x'

Price after first discount = 110x – 20% × 110x = Rs. '88x'

Price after second discount = 88x – 10% × 88x

= Rs. '79.2x'

All the options are equally good for a customer.

Hence, option (d) is correct.

Dishonest Shopkeeper:

Substituting an Inaccurate Weight for the Actual Weight

By employing an incorrect weight, denoted as x grams instead of the standard weight for a kilogram, and selling the item at its cost price, the shopkeeper gets a percentage increase in

profit = $\left[\dfrac{1000 - x}{x} - 100\right]$%

Let us understand this concept through a question:

1. **A dishonest dealer professes to sell his goods at cost price but he uses a weight of 750 gm for a one kg weight. Then, find his profit percentage**

   (*a*) $33\dfrac{1}{5}$%   (*b*) $33\dfrac{1}{7}$%

   (*c*) $33\dfrac{1}{3}$%   (*d*) $33\dfrac{1}{9}$%

Sol: **General Method:**

Let us suppose that the shopkeeper bought 1000 gm or 1 kg of goods for 1000 rupees.

Now according to the question he sold 750 gm instead of 1000 gm for 1000 rupees.

Cost price of 750 gm = 750 rupees.

Selling price of 750 gm = 1000 rupees.

Profit = 1000 – 750 = 250 rupees.

Profit % = $\dfrac{250}{750} \times 100 = 33\dfrac{1}{3}$%

By Formula:

The profit percentage when a false weight is used

$= \left[\dfrac{1000 - x}{x} \times 100\right]$%

Here, x = 750

$\Rightarrow \left[\dfrac{1000 - 750}{750} \times 100\right]$%

$\Rightarrow \left[\dfrac{250}{750} \times 100\right]$

$\Rightarrow \dfrac{2500}{75}$% $= \dfrac{100}{3}$% $= 33\dfrac{1}{3}$%

Hence, option (c) is correct.

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Simple and Compound Interest

IMPORTANT FORMULAE

- **Simple Interest (SI)** = $\dfrac{P \times R \times T}{100}$

- **Total Amount = Principal Amount + Simple Interest**
- **Compound Interest (CI) = A – P**

- $A = P \times \left(1 + \dfrac{R}{100}\right)^T$

Where:

'*A*' is the future value of the investment or loan, including interest.

'*P*' is the principal amount (initial investment or loan amount).

'*R*' is the rate of interest (in per annum).

'*T*' is the time in years.

- **When the Rate of Compound Interest is Different for Different Years:**

$$A = P \times \left(1 + \dfrac{R_1}{100}\right) \times \left(1 + \dfrac{R_2}{100}\right) \times \left(1 + \dfrac{R_3}{100}\right) \times ... \times \left(1 + \dfrac{R_n}{100}\right)$$

Here, '*A*' is the amount accumulated after '*n*' years.

'*P*' = Principal amount invested

And $R_1, R_2, ..., R_n$ is the rate of interest of offered in first year, second year and *n*th year respectively.

- **Difference between CI and SI (*P* is principal and *r* is annual rate expressed as decimal):**

  - For the first compounding period (for first year in general), the SI and CI are equal.

  - The difference between SI and CI for first two years, is equal to one year's interest on the interest for one year = Pr^2

  - The difference between SI and CI for the first three years = $Pr^2 (r + 3)$

Let Principal = P, Rate = R% *p.a.*, Time = *n* years, Amount = A	
When interest compounded annually	**When interest compounded quarterly**
$A = P\left(1 + \dfrac{R}{100}\right)^n$	$A = P\left(1 + \dfrac{R/4}{100}\right)^{4n}$
When interest compounded half – yearly	**When interest compounded monthly**
$A = P\left(1 + \dfrac{R/2}{100}\right)^{2n}$	$A = P\left(1 + \dfrac{R/12}{100}\right)^{12n}$

Lets solve some questions based on these concepts:

1. **A principal 'P' becomes 'Q' in 1 year when compounded half yearly with 'R%' annual rate of interest. If the same principal 'P' becomes 'Q' in 1 year when compounded annually with 'S%' annual rate of interest, then which of the following is correct?**

   (UPSC CSAT 2023)

   (*a*) R = S   (*b*) R > S

   (*c*) R < S   (*d*) R ≤ S

Sol:

Case '1':

When rate of interest is 'R%' and interest is compounded half yearly

Effective rate = $\left(\dfrac{R}{2}\right)$ and time = 2 half years

So, $P\left(1 + \dfrac{R}{200}\right)^2 = Q$ …(*i*)

Case '2':

When rate of interest is 'S%' compounded annually.

So, $P\left(1 + \dfrac{S}{100}\right) = Q$ …(*ii*)

From equation (*i*) and (*ii*), we have;

$$P\left(1 + \dfrac{R}{200}\right)^2 = P\left(1 + \dfrac{S}{100}\right)$$

Or, $1 + \left(\dfrac{R^2}{40000}\right) + \left(\dfrac{R}{100}\right) = 1 + \left(\dfrac{S}{100}\right)$

Or, $\left(\dfrac{R^2}{40000}\right) + \left(\dfrac{R}{100}\right) = \left(\dfrac{S}{100}\right)$

Multiplying each term by 40000, we get;

$R^2 + 400R = 400S$

Or, $S = 0.0025R^2 + R$, Thus, $S > R$

Hence, option (c) is correct.

2. **A person bought a refrigerator worth Rs. 22,800 with 12.5% interest compounded yearly. At the end of first year he paid Rs. 8,650 and at the end of second year Rs. 9,125. How much will he have to pay at the end of third year to clear the debt?** **(UPSC CSAT 2018)**

(*a*) Rs. 9,990   (*b*) Rs. 10,000

(*c*) Rs. 10,590   (*d*) Rs. 11,250

Sol: Given, Principal = Rs. 22,800 and Interest rate = 12.5%

After first year,

Interest = 12.5% of Rs. 22,800 = Rs. 2,850

Amount paid after first-year = Rs. 8,650

So, New principal = 22800 + 2850 – 8650 = Rs.17,000

After second year,

Interest = 12.5% of Rs.17,000 = Rs. 2,125

Amount paid after second-year = Rs. 9,125

So, principal will be = 17000 + 2125 – 9125 = Rs. 10,000

After third year,

Interest = 12.5% of 10,000 = Rs. 1,250

To clear the debt, amount required to be paid

= 10000 + 1250 = Rs. 11,250

Hence, option (d) is correct.

3. **As per agreement with a bank, a businessman had to refund a loan in some equal instalments without interest. After paying 18 instalments he found that 60 percent of his loan was refunded. How many instalments were there in the agreement?**

(UPSC CSAT 2014)

(*a*) 22   (*b*) 24

(*c*) 30   (*d*) 33

Sol: Let the total amount to be paid be Rs. '*A*' and the amount of each installment paid be Rs. '*x*'

Total number of installments will be = $\left(\dfrac{A}{x}\right)$

According to the question

$18x = 60\%$ of $A = 0.6A$

$\left(\dfrac{A}{x}\right) = \dfrac{18}{0.6} = 30$ installments

Hence, option (c) is correct.

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CONCEPT OF MOTION

When an object travels from point A to point B, covering a distance (D), it takes a certain amount of time (T) to do so at a specific speed (S).

Expressing the relationship among T, S, and D:

- **Distance (D)** = Speed (S) × Time (T)

- **Time (T)** = $\dfrac{\text{Distance (D)}}{\text{Speed (S)}}$

- **Speed (S)** = $\dfrac{\text{Distance (D)}}{\text{Time (T)}}$

Note: To solve the problem, all the units involved in the calculation must be uniform i.e., either all of them be in meters and seconds or in kilometers and hours.

- 1 km/hr = $\dfrac{5}{18}$ m/sec and 1 m/sec = $\dfrac{18}{5}$ km/hr

RELATIVE SPEED

When two bodies are moving in the same direction:

If the speeds of bodies A and B are S_A and S_B, their relative speed is $S_A - S_B$ or $S_B - S_A$.

When two bodies are moving in the opposite direction:

If the speeds of bodies A and B are S_A and S_B, their relative speed is $S_A + S_B$.

Now Let's solve some questions based on these concepts:

1. **A thief running at 8 km/hr is chased by a policeman whose speed is 10 km/hr. If the thief is 100 m ahead of the policeman, then the time required for the policeman to catch the thief will be**

   (UPSC CSAT 2013)

   (*a*) 2 min   (*b*) 3 min

   (*c*) 4 min   (*d*) 6 min

Sol: Speed of thief = 8 km/hr

Speed of policeman = 10 km/hr

Relative speed of the policeman with respect to the thief

= 10 – 8 = 2 km/hr = $\dfrac{2000 \text{ meter}}{60 \text{ minutes}}$ = $\dfrac{100}{3}$ m/minute

Now, thief is 100 m ahead of policeman so policeman has to cover 100 m.

So the time taken by policeman to catch the thief

= $\dfrac{100}{\dfrac{100}{3}}$ = 3 minutes.

Hence, option (b) is correct.

2. **A person 'X' from a place 'A' and another person 'Y' from a place 'B' set out at the same time to walk towards each other. The places are separated by a distance of 15 km. 'X' walks with a uniform speed of 1.5 km/hr and 'Y' walks with a uniform speed of 1 km/hr in the first hour, with a uniform speed of 1.25 km/hr in the second hour and with a uniform speed of 1.5 km/hr in the third hour and so on.**

   Which of the following is/are correct?

   I. They take 5 hours to meet.

   II. They meet midway between 'A' and 'B'.

   Select the correct answer using the code given below: **(UPSC CSAT 2021)**

   (*a*) I only   (*b*) II only

   (*c*) Both I and II   (*d*) Neither I nor II

Sol: It is given that the distance between places 'A' and 'B' = 15 km.

Speed of 'X' = 1.5 km/hr

So, distance covered by 'X' in 5 hours = 7.5 km

We need to find the distance covered by 'Y' in 5 hours.

Speed of 'Y' in first hour = 1 km/hr

So, Distance covered by 'Y' in first hour = 1 km

Similarly, Distance covered by 'Y' in second hour = 1.25 km

Distance covered by 'Y' in third hour = 1.5 km

Distance covered by 'Y' in fourth hour = 1.75 km

Distance covered by 'Y' in fifth hour = 2 km

So, the total distance covered by 'Y' in 5 hours

= 1 + 1.25 + 1.5 + 1.75 + 2 = 7.5 km

Thus, they take 5 hours to meet and also they will meet in the midway between 'A' and 'B'.

Hence, option (c) is correct.

TRAIN-RELATED CONCEPTS

(i) The length of the train and the bridge, platform, etc. are always added together to get the distance they need to cover to cross the bridge, platform, etc.

(ii) The length of the train is the only factor that determines the distance they need to cover for objects like poles, trees, etc.

Let's try to solve some questions related to trains for better understanding:

1. A train 200 metres long is moving at the rate of 40 kmph. In how many seconds will it cross a man standing near the railway line? (UPSC CSAT 2018)

(a) 12   (b) 15

(c) 16   (d) 18

Sol: Length of train and speed of train is given and we know that the length of the train is the only factor that determines the distance they need to cross the man.

Speed of the train $= 40$ kmph $= 40 \times \left(\dfrac{5}{18}\right) = \left(\dfrac{100}{9}\right)$ m/s

Time required for a 200 m long train to cross a man

standing on a platform $= \dfrac{\text{length of train}}{\text{speed of train}} = \dfrac{200}{\left(\dfrac{100}{9}\right)}$

$= 18$ second

Hence, option (d) is correct.

2. A train of length 300 m crosses a bridge of length 200 m in 20 seconds. What is the speed of the train?

(a) 75 km/h   (b) 65 km/h

(c) 72 km/h   (d) 90 km/h

Sol: Total distance $= 300 + 200 = 500$ m

Time $= 200$ seconds

Speed of train $= \dfrac{500}{20} = 25$ m/s $= 25 \times \dfrac{18}{5} = 90$ km/h

Hence, option (d) is correct.

BOAT AND STREAM

- **Speed of a boat in still water:** When a river is still and the boat is moving with its own speed.

- **Upstream Speed:** The speed of a boat when it is moving against the direction of current of the river.

- Upstream speed of a boat = Speed of boat in still water – Speed of stream or current

- **Downstream Speed:** The speed of a boat when it is moving along the direction of current of the river.

- Downstream Speed = Speed of boat in still water + Speed of stream or current

Let the speed of a boat in still water be 'b' km/h and speed of stream = 's' km/h. Then,

- Upstream speed of boat = $(b - s)$

- Downstream speed of boat = $(b + s)$

- Speed of boat in still water = b

  $= \dfrac{(\text{Downstream speed} + \text{Upstream speed})}{2}$

- Speed of stream = s

  $= \dfrac{(\text{Downstream speed} - \text{Upstream speed})}{2}$

Now let's solve some questions based on these concepts:

1. A man takes half time in rowing a certain distance downstream than upstream. What is the ratio of the speed in still water to the speed of current?

(UPSC CSAT 2020)

(a) 1:2   (b) 2:1

(c) 1:3   (d) 3:1

Sol: Let the speed of man in still water be 'x' km/h and the speed of current be 'y' km/h.

Let the total distance be 'D' km.

So, Downstream speed $= (x + y)$ km/h

And, Upstream speed $= (x - y)$ km/h

According to the question,

Time taken in rowing upstream

$= 2 \times$ Time taken in rowing downstream

Or, $\dfrac{D}{(x-y)} = 2 \times \left[\dfrac{D}{x+y}\right]$

Or, $x + y = 2x - 2y$

Or, $x = 3y$

Thus, $x : y = 3{:}1$

Hence, option (d) is correct.

2. A boat can move at 10 km/h in still water (i.e., when water is not flowing). The speed of the stream of the river is 2 km/h. A boat takes 80 minutes to go from point A to another point B and return to the same point. What is the distance between the two points?

(a) 7.5 km   (b) 6.5 km

(c) 6.4 km   (d) 7.8 km

Sol: Downstream speed of boat

$= 10 + 2 = 12$ km/h

Upstream speed of boat $= 10 - 2 = 8$ km/h

Therefore,

$\dfrac{\text{Downstream speed}}{\text{Upstream speed}}$

$= \dfrac{\text{Upstream time}}{\text{Downstream time}}$

$\dfrac{12}{8} = \dfrac{3}{2} = \dfrac{\text{Time taken in upstream direction}}{\text{Time taken in downstream direction}}$

So, time taken in upstream direction: time taken in downstream direction = 3:2

Thus, Time taken in downstream = $\dfrac{2}{2+3} \times 80 = 32$ min

Time taken in upstream = $\dfrac{3}{2+3} \times 80 = 48$ min

Distance between two points = downstream speed × downstream time

$= \dfrac{12 \times 32}{60} = 6.4$ km

Distance between two points = upstream speed × upstream time

$= \dfrac{8 \times 48}{60} = 6.4$ km

Hence, option (c) is correct.

RACE BASED QUESTIONS

1. **'X', 'Y' and 'Z' are three contestants in a race of 1000 m. Assume that all run with different uniform speeds. 'X' 'gives 'Y' a start of 40 m and 'X' gives 'Z' a start of 64 m. If 'Y' and 'Z' were to compete in a race of 1000 m, how many metres start will 'Y' give to 'Z'?**
(UPSC CSAT 2019)

   (a) 20   (b) 25
   (c) 30   (d) 35

Sol: In the 1000 m race, 'X' gives 'Y' 40 m start. That means 'Y' starts the race 40 m ahead of 'X'

So, distance covered by 'Y' in the time 'X' covers 1000 metres = 1000 – 40 = 960 metres

So, ratio of speeds of 'X' and 'Y' = Distance covered by 'X' : Distance covered by 'Y' = 1000:960 = 25:24

In the 1000 m race, 'X' gives 'Z' a '64' m start. That means 'Z' starts race 64 m ahead of 'X'

So, distance covered by 'Z' in the time 'X' covers 1000 metres = 1000 – 64 = 936 metres

So, ratio of speeds of 'X' and 'Z' = Distance covered by 'X' : Distance covered by 'Z' = 1000:936 = 125:117

Ratio of speeds of 'X', 'Y' and 'Z' = (25 × 5) : (24 × 5) : 117 = 125:120:117

Thus, ratio of distance covered by 'X', 'Y' and 'Z' = 125:120:117

Therefore, the start which 'Y' can give 'Z' in a race of 100 metres = $\left(\dfrac{120-117}{120}\right) \times 1000 = 25$ metres

Hence, option (b) is correct.

2. **'A' and 'B' walk around a circular park. They start at 8 a.m. from the same point in the opposite directions. 'A' and 'B' walk at a speed of 2 rounds per hour and 3 rounds per hour respectively. How many times shall they cross each other after 8:00 a.m. and before 9:30. a.m.?**
(UPSC CSAT 2016)

   (a) 7   (b) 6
   (c) 5   (d) 8

Sol: Given, speed of 'A' = 2 rounds per hour
Speed of 'B' = 3 rounds per hour
Since, 'A' and 'B' are moving in opposite directions
Relative speed of 'A' w.r.t. 'B' = 2 + 3 = 5 rounds/hour
Hence, they will cross each other 5 times in an hour
From 8 a.m. to 9:30 a.m. (i.e., 1.5 hrs duration), they will cross = 5 × 1.5 = 7.5 times
Thus, they will cross each other 7 times in the given duration.

Hence, option (a) is correct.

3. **A worker reaches his factory 3 minutes late if his speed from his house to the factory is 5 km/hr. If he walks at a speed of 6 km/hr, then he reaches the factory 7 minutes early. The distance of the factory from his house is:**
(UPSC CSAT 2014)

   (a) 4 km   (b) 5 km
   (c) 6 km   (d) 7 km

Sol: Let the distance between house and factory is 'D' km and the actual time taken to reach the factory is 't' minutes.

ATQ;

$\dfrac{D}{5} = \dfrac{t+3}{60}$ ...(i)

And, $\dfrac{D}{6} = \dfrac{t-7}{60}$ ...(ii)

Equating equation (i) and (ii), we get;

$D = \dfrac{5(t+3)}{60} = \dfrac{6(t-7)}{60}$

Or, $5t + 15 = 6t - 42$

Or, $t = 42 + 15 = 57$

Putting 't' in equation (i), we get;

$\dfrac{D}{5} = \dfrac{t+3}{60}$

Or, $\dfrac{D}{5} = \dfrac{57+3}{60}$

So, $D = 5$

Hence, option (b) is correct.

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IMPORTANT CONCEPTS

CONCEPT OF EFFICIENCY

- Efficiency means how much of the work someone can finish in a day. If someone can finish a job in 2 days, they do half of the work each day. Their daily efficiency is 50%.
- If a person takes 4 days to finish the same job, they do one-fourth of the work each day. Their daily efficiency is 25%. It's all about breaking down the work into what can be done in one day.

THE CONNECTION BETWEEN TIME AND EFFICIENCY:

The faster someone works, the less time they need.

For example: If one worker is twice as fast as another, they'll finish the same job in half the time. Speed and time are opposites i.e., when one goes up, the other goes down.

RELATION BETWEEN EFFICIENCIES

2 men finish a work in 3 days. That's a total of $2 \times 3 = 6$ man-days, i.e., 6 men can finish it in 1 day. Similarly, 12 boys take 3 days to finish the same work, making $12 \times 3 = 36$ boy-days, so 36 boys can finish it in 1 day. This means the work of 6 men equals the work of 36 or work of 1 man is equal to work of 6 boys i.e. 1 man is as efficient as 6 boys.

WORK AND WAGES

A group of M_1 men finish a task W_1 in D_1 days, putting in T_1 hours of work each day, and they earn a wage of Rs. R_1. Similarly, another group of M_2 men complete a task W_2 in D_2 days, working T_2 hours per day, and they earn Rs. R_2 as their wage.

$$\frac{M_1 \times D_1 \times H_1}{W_1 \times R_1} = \frac{M_2 \times D_2 \times H_2}{W_2 \times R_2}$$

Now lets see the method for solving questions based on these concepts.

1. **X, Y and Z can complete a piece of work individually in 6 hours, 8 hours and 8 hours respectively. However, only one person at a time can work in each hour and nobody can work for two consecutive hours. All are engaged to finish the work. What is the minimum amount of time that they will take to finish the work?**

   (UPSC CSAT 2024)

   (*a*) 6 hours 15 minutes (*b*) 6 hours 30 minutes
   (*c*) 6 hours 45 minutes (*d*) 7 hours

Sol: To Solve this type of problem we will follow the following steps:

Check what is given:

X can finish the work in 6 hours, so in 1 hour, X does $\frac{1}{6}$ of the work.

Y can finish the work in 8 hours, so in 1 hour, Y does $\frac{1}{8}$ of the work.

Z can also finish the work in 8 hours, so in 1 hour, Z does $\frac{1}{8}$ of the work.

Only one person can work at a time, and no one works for two hours in a row.

What to find: Finish the work as fast as possible by taking turns effectively.

NOTE: Since X is the fastest ($\frac{1}{6} > \frac{1}{8}$), we should use X's efficiency as much as possible. The other two (Y and Z) can help in alternate hours. Let's divide the work among them smartly.

Work of 1st 4 hours:

1. X works for 1 hour: Does $\frac{1}{6}$ of the work.

2. Y works for 1 hour: Does $\frac{1}{8}$ of the work.

3. X works for 1 hour: Does $\frac{1}{6}$ of the work.

4. Z works for 1 hour : Does $\frac{1}{8}$ of the work

Total work done in 4 hours:

$$\frac{1}{6} + \frac{1}{8} + \frac{1}{6} + \frac{1}{8} = \frac{4}{25} + \frac{3}{24} + \frac{4}{25} + \frac{3}{24} = \frac{14}{24} = \frac{7}{12}.$$

So, after 4 hours, $\frac{7}{12}$ of the work is done, and $\frac{5}{12}$ i.e. $\left(1 - \frac{7}{12}\right)$ is left.

Next 2 hours:

X works for 1 hour: Does $\frac{1}{6}$ of the work.

Y works for 1 hour: Does $\frac{1}{8}$ of the work.

Total work done in these 2 hours:

$\dfrac{1}{6}+\dfrac{1}{8} = \dfrac{4}{24}+\dfrac{3}{24} = \dfrac{7}{24}$.

Now, the total work completed is:

$\dfrac{7}{12}+\dfrac{7}{24} = \dfrac{14}{24}+\dfrac{7}{24} = \dfrac{21}{24} = \dfrac{7}{8}$.

Work left = $1-\dfrac{7}{8} = \dfrac{1}{8}$

Now X will work and efficiency of X is $\dfrac{1}{6}$.

So, time taken by X to finish the work = 6 hours

Time taken by X to complete $\dfrac{1}{8}$ of the work = $\dfrac{6}{8}$ hours = 45 minutes.

Total time = 6 hours + 45 minutes = 6 hours 45 minutes.

Alternate solution:

Let total amount of work is 24 units.

Efficiency of 'X' = $\left(\dfrac{24}{6}\right)$ = 4 units per hour

Efficiency of 'Y' = $\left(\dfrac{24}{8}\right)$ = 3 units per hour

Efficiency of 'Z' = $\left(\dfrac{24}{8}\right)$ = 3 units per hour

Since, only one person work at a time and no body can work for two consecutive hours.

Thus, work completed by 'A' and 'B' in 6 hours = 4 + 3 + 4 + 3 + 4 + 3 = 21 units

Remaining work = 24 – 21 = 3 units

3 units of work will be completed by 'X' in $\left(\dfrac{3}{4}\right)$ hour i.e. 45 minutes.

So, required time = 6 hours 45 minutes.

Hence, option (c) is correct.

2. **A man completes $\left(\dfrac{7}{8}\right)$th part of a job in 21 days. How many more days will it take him to finish the job if quantum of work is further increased by 50%.**
   (UPSC CSAT 2021)
   (a) 24
   (b) 21
   (c) 18
   (d) 15

Sol: Time taken by the man to complete the whole work

$= \dfrac{21}{\frac{7}{8}}$ = 24 days

Let the total amount of work = 24 units

So, efficiency of the man $=\dfrac{24}{24}$ = 1unit per day

New amount of work = 1.50 × 24 = 36 units

Extra amount of work to be completed

$= 36 – 24 \times \dfrac{7}{8}$ = 15 units

Time taken by the man to complete 15 units of work

$= \dfrac{15}{1}$ = 15 days

Alternate Method:

Total initial units of work = 8 units

Out of which, 7 units are completed in 21 days.

So, the number of days that man takes in completing

one unit of work $=\dfrac{21}{7}$ = 3 days

Now, amount of new work = 8 + 50% of 8 = 8 + 4 = 12 units

Work remaining = 12 – 7 = 5 units

So, the time taken by the man to complete the rest of the work = 5 × 3 = 15 days

Hence, option (d) is correct.

3. **'P' works thrice as fast as 'Q', whereas 'P' and 'Q' together can work four times as fast as 'R'. If 'P', 'Q' and 'R' together work on a job, in what ratio should they share the earnings?** **(UPSC CSAT 2017)**
   (a) 3:1:1
   (b) 3:2:4
   (c) 4:3:4
   (d) 3:1:4

Sol: Let the efficiencies of 'P', 'Q' and 'R' be 'p' units/day, 'q' units/day and 'r' units/day, respectively.

Given; $p = 3q$ ...(i)

and $p + q = 4r$

So, $3q + q = 4r$ [putting $p = 3q$]

Or, $4q = 4r$

Or, $q = r$

Thus, the ratio in which they will share their earnings is proportionate to their respective efficiencies

$= p:q:r = 3q:q:q = 3:1:1$

Hence, option (a) is correct.

PIPES AND CISTERNS

Pipes and Cisterns play a role in filling or emptying tanks, reservoirs, or cisterns. An Inlet refers to a pipe or tap connected to a tank that fills it, while an Outlet is a pipe or tap linked to a tank that empties it. All the concepts of efficiency and negative work are very useful here.

CONCEPT OF NEGATIVE WORK

If Ram writes 20 pages in an hour but Riya erases 10 of them, Ram is left with only 10 pages done in an hour. This means Riya reduces his work, so he effectively completes 10 pages per hour. Lets understand this through questions:

1. **A tank is half full. Pipe A can fill the tank in 14 min. Pipe B can empty it in 21 min. If both pipes are open, then how long will it take to empty the tank?**

   (a) 21 min   (b) 25 min

   (c) 20 min   (d) 42 min

Sol: Pipe A can fill $\dfrac{1}{14}$ of the tank in 1 min.

Pipe B can empty $\dfrac{1}{21}$ of the tank in 1 min.

So, In 1 minute, part tank emptied = $\dfrac{1}{14} - \dfrac{1}{21} = \dfrac{1}{42}$

In 42 minutes the whole tank is empty.

Since tank is half full, so time taken to empty half filled tank = $\dfrac{1}{2} \times 42 = 21$ minutes.

Hence, option (a) is correct.

2. **Two pipes 'A' and 'B' can independently fill a tank completely in 20 minutes and 30 minutes, respectively. If both the pipes are opened simultaneously, how much time will they take to fill the tank completely?**
   (UPSC CSAT 2015)

   (a) 10 minutes

   (b) 12 minutes

   (c) 15 minutes

   (d) 25 minutes

Sol:

Pipe A can fill $\dfrac{1}{20}$ of the tank in 1 min.

Pipe B can fill $\dfrac{1}{30}$ of the tank in 1 min.

In 1 minute, part of tank filled = $\dfrac{1}{20} + \dfrac{1}{30} = \dfrac{5}{60} = \dfrac{1}{12}$

In 1 minute these pipes can fill $\dfrac{1}{12}$ of the tank.

Time taken to both the pipes to fill the tank = 12 minutes.

Alternate solution:

Given,

Time taken by A and B to completely fill a tank is 20 minutes and 30 minutes, respectively

Let the total capacity of tank be = 60 litres (LCM of 20 and 30 = 60)

Efficiency of pipe 'A' = $\dfrac{60}{20}$ = 3 litres/min

Efficiency of pipe 'B' = $\dfrac{60}{30}$ = 2 litres/min

Combined efficiency of pipe 'A' and 'B' = 3 + 2 = 5 litres/min

Required time = $\dfrac{60}{5}$ = 12 mintues

Hence, option (b) is correct.

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Permutation and Combination

FACTORIAL NOTATION

The continued product of first '*n*' natural numbers is called 'n-factorial' and is denoted as *n*!.

$n! = n(n-1)(n-2)\ldots\ldots 3.2.1$

Points to Remember

- $0! = 1$
- Factorials of only natural numbers are defined.
- $n! = n(n-1)!$ i.e. $6! = 6 \times 5!$ or $8! = 8 \times 7!$
- **Remember:** $0! = 1$, $1! = 1$, $2! = 2$, $3! = 6$, $4! = 24$, $5! = 120$, $6! = 720$

FUNDAMENTAL PRINCIPLE OF COUNTING

In FPC there are two rule *i.e.*, addition and multiplication, the addition rule is used when events are independent while the multiplication rule is used when events are dependent.

(a) Multiplication Rule: If there are two tasks such that one of them can be completed in '*m*' different ways following which another task can be completed in '*n*' different ways, then the two tasks in succession can be completed in (m × n) ways. This rule can be further extended to any number of tasks.

Let us understand this concept through a question:

1. A flag has to be designed with 4 horizontal strips using some or all of the colours red, green and yellow. What is the number of different ways in which this can be done so that no two adjacent stripes have the same colour? **(UPSC CSAT 2023)**

(*a*) 12 (*b*) 18
(*c*) 24 (*d*) 36

Sol: Let's consider the possibilities for each stripe:

For the first stripe, we have three colour options – (red, green, yellow). So, first stripe can be painted in three different ways.

For the second stripe, we need to choose a colour different from the first stripe.

So, we have 2 colour options for stripe '2'.

For the third stripe, we need to choose a colour different from the second stripe.

So, we have 2 colour options for stripe '3'.

For the fourth stripe, we need to choose a colour different from the third stripe.

So, we have 2 colour options for stripe '4'.

Stripe '1' - can be coloured in 3 ways
Stripe '2' - can be coloured in 2 ways
Stripe '3' - can be coloured in 2 ways
Stripe '4' - can be coloured in 2 ways

So, total number of possible ways to design the flag $= 3 \times 2 \times 2 \times 2 = 24$

Hence, option (c) is correct.

(b) Addition Rule: If there are two events that cannot happen at the same time (mutually exclusive events), and if one event can occur in 'm' ways and another event can occur in 'n' ways, then there are (m + n) ways for either of the two events to occur.

Let us understand this concept through a question:

2. Raj has ten pairs of red, nine pairs of white and eight pairs of black shoes in a box. If he randomly picks shoes one by one (without replacement) from the box to get a red pair of shoes to wear, what is the maximum number of attempts he has to make?

(UPSC CSAT 2023)

(*a*) 27 (*b*) 36
(*c*) 44 (*d*) 45

Sol: Raj has ten pairs of red, nine pairs of white and eight pairs of black shoes in a box.

So, he has 20 red shoes, 18 white shoes, and 16 black shoes.

To find the maximum number of attempts we need to visualize the worst-case scenario.

Let us suppose that Raj draws all white and black shoes, which amounts to $18 + 16 = 34$ shoes.

Now, if he picks any two shoes, they are certainly going to be red.

However, we also need to ensure that the red shoes picked by him make a pair, i.e. there should be one red shoe for left foot and one red shoe for right foot.

For this to happen we need to pick 11 red shoes.

So, maximum number of attempts to get a red pair of shoes $= 34 + 11 = 45$

Hence, option (d) is correct.

PERMUTATION VS COMBINATION

Permutation refers to the arrangement or ordering of a set of objects in a specific sequence or order.

While **combination** refers to the selection of a subset of objects from a larger set, where the order of selection does not matter.

Consider a set of three objects: A, B, and C. We want to count the number of ways to select two objects from this set.

Permutations: If we are interested in the permutations of two objects from this set, **we consider the order**. The permutations would be:

- AB (A selected first, B selected second)
- BA (B selected first, A selected second)
- AC (A selected first, C selected second)
- CA (C selected first, A selected second)
- BC (B selected first, C selected second)
- CB (C selected first, B selected second)

Combinations: If we are interested in the combinations of two objects, **we don't consider the order**. The combinations would be: AB, AC and BC (Here AB and BA are considered the same as the order doesn't matter)

COUNTING FORMULA FOR COMBINATION

Selecting 'r' things out of 'n' distinct things is denoted as: ^nC_r

or $C(n, r)$ and $^nC_r = \dfrac{n!}{r!(n-r)!} = \dfrac{n(n-1)(n-2)...(n-r+1)}{r!}$

For example: Selecting a playing 11 from a team of 15 players is $^{15}C_{11}$.

The Following Geometrical Results must be Remembered

1. Given a set of n points in a plane, with the condition that no three of them are collinear:

   (a) Number of straight lines that can be formed:

   Number of ways by which we can select any two points gives the total number of straight lines = $^nC_2$.

   (b) Number of triangles that can be formed:

   Number of ways by which we can select any three non-collinear points gives total number of triangles = $^nC_3$

   (c) The total number of diagonals in a polygon with n sides:

   Number of diagonals = Total lines – (number of sides of polygon) = $^nC_2 - n$

2. Given a set of n points in a plane, with the condition that m points of them are collinear:

   (d) Number of straight lines that can be formed:

   Number of ways by which we can select any two points gives the total number of straight lines = $^nC_2 - ^mC_2 + 1$

   (e) Number of triangles that can be formed:

   Number of ways by which we can select any three non-collinear points gives total number of triangles = $^nC_3 - ^mC_3$

1. **A selection is to be made for one post of Principal and two posts of Vice-Principal. Amongst the six candidates called for the interview, only two are eligible for the post of Principal while they all are eligible for the post of Vice-Principal. The number of possible combinations of selectees is** **(UPSC CSAT 2015)**

   (a) 4 (b) 12

   (c) 18 (d) None of the above

Sol: Given, a selection is to be made for one post of Principal and two posts of Vice-Principal

   Number of candidate for the post of principal = 2

   Number of candidate for the post of vice principal = 5 [When out of the six candidates one is selected for the principal position]

   Number of Possible combinations = $^2C_1 \times ^5C_2$

   $= 2 \times 10 = 20$

   Hence, option (d) is correct.

2. **How many diagonals can be drawn by joining the vertices of an octagon?** **(UPSC CSAT 2018)**

   (a) 20 (b) 24

   (c) 28 (d) 64

Sol: Using the formula,

   Number of diagonals of n-sided polygon

   = Total lines – (number of sides of polygon)

   = $^nC_2 - n$

   Here, $n = 8$ (for Octagon)

   Diagonals = $^8C_2 - 8 = 28 - 8 = 20$.

   Hence, option (a) is correct.

LINEAR PERMUTATIONS:

The number of permutations of n different things taken r at a time is ^nP_r, where

$$^nP_r = \dfrac{n!}{(n-r)!} = n(n-1)(n-2)\ldots(n-r+1)$$

where 'n' is the total number of items, 'r' is the number of items you're arranging.

Suppose you have a set of 7 books, and you want to arrange 3 of them on a bookshelf in a specific order. In how many different ways can you arrange these 3 books?

Using the permutation formula: $^7P_3 = \dfrac{7!}{(7-3)!} = \dfrac{7!}{4!}$
$= \dfrac{7 \times 6 \times 5 \times 4!}{4!} = 210.$ or simply as $7 \times 6 \times 5 = 210$.

Note: The number of all permutations of 'n' distinct objects taken all at a time without repetition = ^nP_n = n!

Let us understand this concept through a question:

1. **How many 3-digit natural numbers (without repetition of digits) are there such that each digit is odd and the number is divisible by 5?** (UPSC CSAT 2022)

   (*a*) 8

   (*b*) 12

   (*c*) 16

   (*d*) 24

Sol: Let the number be '*ABC*'

Since, the digits are odd therefore the three digits must be from amongst 1, 3, 5, 7, and 9.

Moreover, since the number is divisible by '5', i.e. the unit's digit i.e '*C*' must be '5'.

Remaining digits left = 1,3,7,9.

The number of ways we can fill the first two digits from

amongst 4 distinct digits = $^4P_2 = \dfrac{4!}{2!} = 4 \times 3 = 12.$

Hence, option (b) is correct.

2. **In a question paper there are five questions to be attempted and answer to each question has two choices - True (*T*) or False (*F*). It is given that no two candidates have given the answers to the five questions in an identical sequence. For this to happen the maximum number of candidates is:** (UPSC CSAT 2016)

   (*a*) 10

   (*b*) 18

   (*c*) 26

   (*d*) 32

Sol: Total number of Question = 5

And each question can be answered in 2 ways

Thus, total number of ways to answer the questions $= 2^5 = 32$

Maximum number of candidates will be equal to the total number of ways to answer the questions such that no two students would have answered all the questions in the same sequence.

Maximum number of candidates = 32

Hence, option (d) is correct.

PERMUTATION OF ALIKE OBJECTS

When you want to find the number of permutations of a set of objects, some of which are alike, you can use the concept of permutations with repetition. The formula for permutations of alike objects is:

$$P(n; n_1, n_2, n_3, ..., n_k) = \dfrac{n!}{n_1! \, n_2! \, n_3! \ldots \ldots n_k!}$$

where P represents the number of permutations, n is the total number of objects in the sets n_1, n_2, n_3,..., n_k of similar type.

Let us understand this concept through a question:

1. **Using 2, 2, 3, 3, 3 as digits, how many distinct numbers greater than 30000 can be formed?** (UPSC CSAT 2021)

   (*a*) 3

   (*b*) 6

   (*c*) 9

   (*d*) 12

Sol: For any number to be greater than 30000, it must start with the digit 3.

Also, as only 5 digits are given to us and all the digits must be used.

3	_	_	_	_

The 4 blanks have to be filled by two 2's and two 3's.

Number of ways to do so $= \dfrac{4!}{2! \times 2!} = 6$

These numbers are: 33322, 33232, 33223, 32332, 32323, and 32233.

Hence, option (b) is correct.

2. **How many words can be formed with the letters of the word MATHEMATICS by rearranging them?**

   (*a*) $\dfrac{11!}{2!2!}$

   (*b*) $\dfrac{11!}{2!}$

   (*c*) $\dfrac{11!}{2!2!2!}$

   (*d*) 11!

Sol:

Since there are 2 M's, 2 A's and 2 T's.

$\therefore$ Required number of ways are

$\dfrac{11!}{2!2!2!}.$

Hence, option (c) is correct.

CIRCULAR PERMUTATIONS

Circular permutation deals with the arrangement of objects, individuals, or elements in a circular or closed-loop fashion. Unlike linear permutation, circular permutation involves arranging objects in a circle. This means that there are no distinct endpoints (starting points).

Circular Permutation Formula: The number of ways to arrange 'n' distinct objects in a circle is given by

(n - 1)! when rotations are considered identical. Remember that here clockwise and anti-clockwise arrangements are considered different.

Note: If we arrange flowers or garland beads in a necklace then there is no distinction between clockwise & anticlockwise direction. So the formula becomes $\dfrac{(n-1)!}{2}$

Let us understand this concept through a question:

1. **Six friends are sitting around a circular table to play a board game. How many different seating arrangements are possible if two of the friends, Alice and Bob, insist on sitting next to each other?**

   (a) 24   (b) 36

   (c) 48   (d) 64

Sol: Here, treat Alice and Bob as a single entity because they must sit next to each other. This reduces the problem to arranging 5 entities (Alice & Bob, and the other 4 friends) in a circle.

Now, we can use circular permutation to arrange these 5 entities. The formula for circular permutation is (n – 1)!, where 'n' is the number of entities.

In this case, 'n' is 5.

Number of Arrangements = [(5 – 1)!] × 2! = 4! × 2! = 48. Here 2! is also multiplied for the arrangement of Alice and Bob (AB, BA).

Hence, option (c) is correct.

STRING METHOD

If you want to consider arrangements where some specific items are together, you can treat these specific items as a single unit. This reduces the problem of arranging the rest of the items, including the combined unit.

Let's understand this with an example:

Suppose you have 5 friends, *A*, *B*, *C*, *D*, and *E*, and you want to arrange them in a row of chairs. However, you specifically want A and B to sit together.

Treat AB as a Single Unit (tied with a string): Consider *A* and *B* together as a single unit, denoted as AB. Now, you have four "units" to arrange: *AB*, *C*, *D*, and *E*.

Arrangements within the AB Pair: Within the *AB* pair, you can arrange *A* and B in two ways: AB or BA.

Total Arrangements

- Now, arrange the four units (AB, C, D, E) in a row. The number of arrangements is 4! = 24.
- However, within the AB pair, you have 2!(=2) ways to arrange A and B.
- So, the total number of arrangements is 24 × 2 = 48

GAP METHOD

If you want to consider arrangements of some items when no two of some specified items are together.

To arrange 'n' different things in such a way that no two of the 'r' things are together, then we arrange the 'r' things in gaps created by arranging remaining (*n–r*) things.

Suppose you have a group of 3 boys (B1, B2, B3) and 3 girls (G1, G2, G3), and you want to arrange them such that no two boys are next to each other.

Arrangement of girls: First of all, arrange 3 girls for which there is no restriction. This can be done in 3! ways.

Creation of gaps: Arrangement of 3 girls in a line creates 4 gaps. One such arrangement is (_G1_G2_G3_)

Arrangement of boys in gaps: Now select 3 gaps out of 4 (in $^4C_3$ ways) for 3 boys and arrange them in gaps in 3! ways.

Total Arrangements: $3! \cdot {}^4C_3 \cdot 3! = 6 \times 4 \times 6 = 144$ ways.

Let us understand this concept through a question:

1. **In how many ways 7 men and 7 women can be seated around a round table such that no two women can sit together**

   (a) $(7!)^2$

   (b) $7! \times 6!$

   (c) $(6!)^2$

   (d) $7!$

Sol: Firstly 7 men can be seated in a circle in 6! ways. Now no two women are to sit together and as such the 7 women are to be arranged in seven empty seats (gaps) between two consecutive men. So 7 women can be seated in 7 gaps in 7! ways.

Therefore, total number of ways = 6! × 7!

Hence, option (b) is correct.

2. **If 2 boys and 2 girls are to be arranged in a row so that the girls are not next to each other, how many possible arrangements are there?** **(UPSC CSAT 2017)**

   (a) 3   (b) 6

   (c) 12   (d) 24

Sol: Let us represent two boys as 'B1' and 'B2' while two girls as 'G1' and 'G2'

The possibilities of arrangements are:

G1 B1 G2 B2	B1 G2 B2 G1	G1 B1 B2 G2
G1 B2 G2 B1	B2 G2 B1 G1	G1 B2 B1 G2
G2 B1 G1 B2	B1 G1 B2 G2	G2 B1 B2 G1
G2 B2 G1 B1	B2 G1 B1 G2	G2 B2 B1 G1

Hence, option (c) is correct.

3. **How many triplets (x, y, z) satisfy the equation $x + y + z = 6$, where 'x', 'y' and 'z' are natural numbers ?**

   (UPSC CSAT 2019)

   (a) 4   (b) 5
   (c) 9   (d) 10

Ans: We will get 3 digits sum as 6 in three cases. $(1, 2, 3)$, $(1, 1, 4)$ and $(2, 2, 2)$

Case-1:

Digits $(1, 2, 3)$ can be rearranged in 3! ways = 6 ways

Case-2:

Digits $(1, 1, 4)$ can be rearranged in

$\dfrac{3!}{2!}$ ways = 3 ways

Case-3:

Digits $(2, 2, 2)$ can be rearranged in 1 way

Required number of triplets = $6 + 3 + 1 = 10$

We get 10 sets of triplets as:

$(1, 2, 3), (1, 3, 2), (2, 1, 3), (2, 3, 1), (3, 1, 2), (3, 2, 1),$
$(1, 1, 4), (1, 4, 1), (4, 1, 1)$ and $(2, 2, 2)$

Hence, option (d) is correct.

❖   ❖   ❖

$$\text{Probability } = \frac{\text{Number of favourable outcomes}}{\text{Total number of outcomes}}$$

PROBABILITY RELATED WITH TOSSING OF COINS

When working with coins in probability, we often consider a fair coin, which is a coin with two equally likely outcomes: Heads (H) and Tails (T).

- **Tossing a Single Coin:**

  Total number of possible outcomes = 2

  Sample Space = $\{H, T\}$

- **Tossing Two Coins:**

  Total number of possible outcomes = 2^2 = 4

  Sample Space = $\{HH, HT, TH, TT\}$

- **Tossing Three Coins:**

  Total number of possible outcomes = 2^3 = 8

  Sample Space = $\{HHH, HHT, HTH, HTT, THH, THT, TTH, TTT\}$

- When flipping a fair coin multiple times, each flip is independent of the others. The probability of getting heads or tails remains 0.5 on each flip.

  For Example: P(one head) = P(HTT) + P(THT) + P(TTH)

  = (0.5)(0.5)(0.5) + (0.5)(0.5)(0.5) + (0.5)(0.5)(0.5)

  $= \frac{1}{8} + \frac{1}{8} + \frac{1}{8} = \frac{3}{8}$

PROBABILITY RELATED WITH ROLLING OF DICE

Standard six-sided dice are commonly used in probability scenarios, and they have six equally likely outcomes: 1, 2, 3, 4, 5, and 6.

- Rolling a single die

  Total number of possible outcomes = 6

  Sample Space = $\{1, 2, 3, 4, 5, 6\}$

- Rolling of two dice

  Total number of possible outcomes = 6^2 = 36

Sample Space = $\{(1, 1), (1, 2), (1, 3), (1, 4), (1, 5), (1, 6),$
$(2, 1), (2, 2), (2, 3), (2, 4), (2, 5), (2, 6),$
$(3, 1), (3, 2), (3, 3), (3, 4), (3, 5), (3, 6),$
$(4, 1), (4, 2), (4, 3), (4, 4), (4, 5), (4, 6),$
$(5, 1), (5, 2), (5, 3), (5, 4), (5, 5), (5, 6),$
$(6, 1), (6, 2), (6, 3), (6, 4), (6, 5), (6, 6)\}$

When rolling multiple dice, outcome of each die is independent of each other.

The probability of a specific combination of outcomes is calculated by multiplying the probabilities of each die's outcome. For example, the probability of rolling a 4 on one die and a 2 on another die i.e., (4, 2) is $= \frac{1}{6} \times \frac{1}{6} = \frac{1}{36}$

- Probability of getting a doublet (Same number)

  $= \frac{6}{36}$

- **Sum of Dice Rolls:** For example, there are multiple ways to obtain a sum of 7: (1, 6), (2, 5), (3, 4), (4, 3), (5, 2), and (6, 1). Each of these combinations has a probability of $\frac{1}{6} \times \frac{1}{6} = \frac{1}{36}$.

  Therefore, the probability of rolling a sum of 7 is $= \frac{6}{36} = \frac{1}{6}$.

PROBABILITY RELATED WITH A DECK OF WELL SHUFFLED CARD

A standard deck of 52 playing cards consists of four suits [hearts, diamonds, clubs, and spades), each with 13 ranks numbers from 2 to 10, and face cards (Jack, Queen, King), and an Ace].
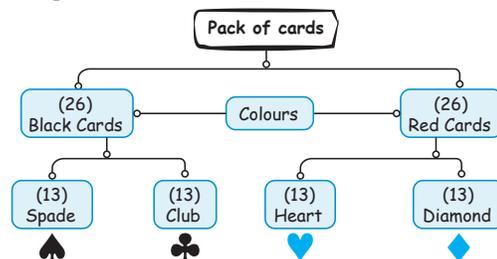
- Total cards: 52
- Red Cards = Black Cards = 26 each
- Hearts = Diamond = Spade = Clubs = 13 cards each
- Total King = Total Queen = Total Jack = Total Ace = 4 each
- Face Cards : King (4) + Queen (4) + Jack (4) = 12

The probability of drawing a specific card (e.g., the Ace of Spades) from a standard deck is, P(Ace of Spades) = $\dfrac{1}{52}$

The probability of drawing a card of a specific rank (e.g., any Ace) is P(Any Ace) = $\dfrac{4}{52} = \dfrac{1}{13}$, since there are four Aces in the deck.

Addition Theorem

- $P(A \cup B) = P(A \text{ or } B) = P(A) + P(B) - P(A \cap B)$
- If A & B are mutually exclusive then $P(A \cap B) = 0$ and so $P(A \cup B) = P(A) + P(B)$

  In case of three events, 'A', 'B' and 'C'

Let us solve some questions to understand these concepts:

1. **A bag contains 15 red balls and 20 black balls. Each ball is numbered either '1' or '2' or '3'. Out of the total number of red balls 20% are numbered '1' while 40% are numbered '3'. Similarly, among the black balls, 45% are numbered '2' and 30% are numbered '3'. A boy picks a ball at random. He wins if the ball is red and numbered '3' or if it is black and numbered '1' or '2'. What are the chances of his winning?**

   (UPSC CSAT 2018)

   (a) $\dfrac{1}{2}$ (b) $\dfrac{4}{7}$

   (c) $\dfrac{5}{9}$ (d) $\dfrac{12}{13}$

Sol: Total number of balls = Number of red balls + Number of black balls = 15 + 20 = 35

Number of red balls which are numbered '1'

= 20% of 15 = 3

Number of red balls which are numbered '3'

= 40% of 15 = 6

Number of red balls which are numbered '2'

= 15 – (3 + 6) = 6

Number of black balls which are numbered '2'

= 45% of 20 = 9

Number of black balls which are numbered '3'

= 30% of 20 = 6

Number of black balls which are numbered '1'

= 20 – (9 + 6) = 5

Condition for winning: if the ball is red and numbered '3' or if it is black and numbered '1' or '2'

So, total number of favourable outcomes = Number of red balls which are numbered '3' + Number of black balls which are numbered '1' + Number of black balls which are numbered '2' = 6 + 5 + 9 = 20

Required probability = $\left(\dfrac{20}{35}\right) = \left(\dfrac{4}{7}\right)$

Hence, option (b) is correct.

2. **A round archery target of diameter 1 m is marked with four scoring regions from the centre outwards as red, blue, yellow and white. The radius of the red band is 0.20 m. The width of all the remaining bands is equal. If archers throw arrows towards the target, what is the probability, that the arrows fall in the red region of the archery target?   (UPSC CSAT 2016)**

   (a) 0.40   (b) 0.20

   (c) 0.16   (d) 0.04

Sol:
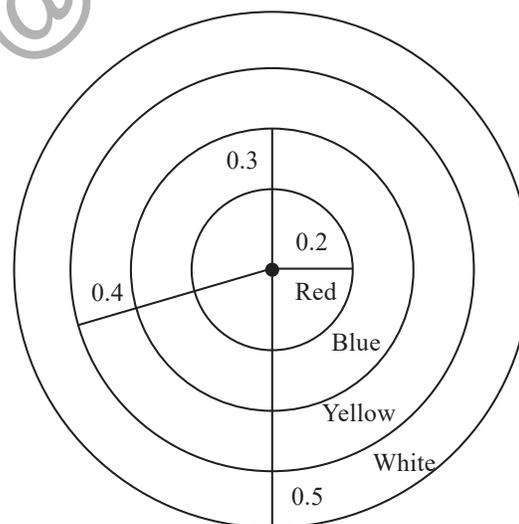
Diameter = 1 m

So, the radius = $\left(\dfrac{1}{2}\right) = 0.5$ m

Area of red band = $\pi r^2 = \pi (0.2)^2 = 0.04\pi\, m^2$

Total target area = $\pi r^2 = \pi (0.5)^2 = 0.25\pi\, m^2$

Required probability = $\dfrac{\text{Area of red band}}{\text{Total target area}}$

$= \dfrac{0.04\pi}{0.25\pi} = 0.16$

Hence, option (c) is correct.

13 Sequence and Series

ARITHMETIC PROGRESSION

- n^{th} term of an AP is given by $T_n = a + (n-1)d$
  where 'a' is first term and 'd' is common difference
- Sum of the First 'n' Terms of an AP

  $$S_n = \frac{n}{2} \times \left[2a + (n-1)d \right] = \frac{n}{2}\left[a + T_n \right]$$

  = Number of terms × (Average of first and last term)

- **Arithmetic Mean (Mean or Average) (AM):**
  If three terms are in AP, then the middle term is called the AM between the other two terms i.e. if 'a', 'b', 'c' are in AP, then 'b' is AM of 'a' & 'c'.

- AM for any 'n' numbers $a_1, a_2, ..., a_n$ is

  $$= \frac{a_1 + a_2 + a_3 ....+ a_n}{n}$$

GEOMETRIC PROGRESSION

- n^{th} term of a GP is given by $T_n = ar^{n-1}$
  Where, **'a'** is first term of GP and **'r'** is the common ratio.
- The sum of the first 'n' terms of a geometric progression (GP), often denoted as S_n is:

  $$S_n = \frac{a(1-r^n)}{1-r}, \text{ when } r < 1$$

  $$S_n = \frac{a(r^n-1)}{r-1}, \text{when } r > 1$$

- Sum of infinite terms of a GP $= \frac{a}{1-r}$, when $r < 1$
- **Geometric Mean (GM)**
  If three terms are in GP then the middle term is called the GM of the other two terms.

  So, if 'a', 'b' and 'c' are in GP, then 'b' is GM of 'a' & 'c'.

  If 'a' and 'c' are both positive, then $b = \sqrt{ac}$

  And if 'a' and 'c' are both negative, then $b = -\sqrt{ac}$

- GM for any 'n' positive numbers $a_1, a_2, ..., a_n$ is

  $$= \sqrt[n]{a_1 \times a_2 \times a_3 \times a_4 \times ....\times a_n}$$

  $$= (a_1 \times a_2 \times a_3 \times a_4 \times ....\times a_n)^{\frac{1}{n}}$$

HARMONIC PROGRESSION

- n^{th} Term of HP

  General HP can be considered as $\frac{1}{a}$, $\frac{1}{a+d}$, ...

  where 'a' and 'd' are first term and common difference of corresponding AP.

  Thus, n^{th} term of the HP, $T_n = \frac{1}{a+(n-1)d}$

- **Harmonic Mean (HM)**
  If three terms are in HP, then the middle term is called the HM between the other two terms.

  So, if 'a', 'b', 'c' are in HP, then 'b' is HM of 'a' & 'c'.

  Also, $b = \frac{2ac}{a+c}$

- If $a_1, a_2, ... a_n$ are 'n' non-zero numbers then Harmonic Mean (H) of these numbers is given by

  $$\frac{1}{H} = \frac{1}{n} \times \left[\frac{1}{a_1} + \frac{1}{a_2}... ....+ \frac{1}{a_n} \right]$$

RELATIONSHIP BETWEEN AM, GM AND HM

If AM, GM, HM are Arithmetic Mean, Geometric Mean and Harmonic Mean between any two numbers, then

- $AM \geq GM \geq HM$ (equality holds when all terms are equal)
- $(GM)^2 = (AM) \times (HM)$ (i.e., AM, GM, HM are in GP)

Lets solve some questions to understand these concepts:

1. **A sum of Rs. 700 has to be used to give seven cash prizes to the students of a school for their overall academic performance. If each prize is Rs. 20 less than its preceding prize, then what is the least value of the prize?** **(UPSC CSAT 2013)**

   (a) RS. 30   (b) RS. 40

   (c) RS. 60   (d) RS. 80

Sol: Let us take the least prize = x

According to the question each prize is Rs. 20 less than its preceding prize, then the other prizes will be x + 20, x + 40, x + 60, x + 80, x + 100, x + 120.

Now total prize money = Rs.700

So, x + x + 20 + x + 40 + x + 60 + x + 80 + x + 100 + x + 120 = 700

$7x + 420 = 700 \Rightarrow 7x = 280 \Rightarrow x = 40$.

So, the lowest prize is Rs. 40.

Hence, option (b) is correct.

2. **From January 1, 2021, the price of petrol (in Rupees per litre) on m^{th} day of the year is $(80 + 0.1m)$, where 'm' = 1, 2, 3, …, 100 and thereafter remains constant. On the other hand, the price of diesel (in Rupees per litre) on the nth day of 2021 is $(69 + 0.15n)$ for any 'n'. On which date in the year 2021 are the prices of these two fuels equal?** (UPSC CSAT 2021)

  (a) 21st May   (b) 20th May

  (c) 19th May   (d) 18th May

Sol: It is given that Price of the petrol on m^{th} day of the year

= 80 + 0.1 m

Where m = 1 to 100. After which it remains constant.

Price of the petrol on and after 100th day

= 80 + 0.1 × 100 = 80 + 10 = Rs. 90

Now, total number of days till 30th April

= 31 (January) + 28 (February) + 31 (March) + 30 (April)

= 120 days

Let's consider the options.

Option (a):

21st May means 120 + 21 = 141 days

So, price of the diesel on the given day

= 69 + 0.15 × 141 = Rs. 90.15

Therefore, we can say that on 21st May 2021 price of these two fuels is not equal.

Option (b):

20th May means 120 + 20 = 140 days

So, price of the diesel on the given day

= 69 + 0.15 × 140 = Rs. 90

Therefore, we can say that on 20th May 2021, price of these two fuels will be equal.

Hence, option (b) is correct.

3. **The 6th term of GP is 32 and its 8 th term is 128 . Then the common ratio of GP is –**

  (a) – 1   (b) 2

  (c) 4   (d) – 4

Sol: Accordingly, $T_n = ar^{(n-1)}$

$\Rightarrow T_6 = ar^5 = 32$ ...(i)

Similarly $T_8 = ar^{(8-1)}$

$\Rightarrow 128 = ar^7$

Dividing eq. (ii) from eq. (i)

$\Rightarrow r^2 = 4 \Rightarrow r = 2$

Hence, option (b) is correct.

4. **One page is torn from a booklet whose pages are numbered in the usual manner starting from the first page as 1. The sum of the numbers on the remaining pages is 195. The torn page contains which of the following numbers.** (UPSC CSAT 2020)

  (a) 5, 6   (b) 7, 8

  (c) 9, 10   (d) 11, 12

Sol: Let the number of pages in the book be 'n'.

Sum of consecutive numbers from 1 to n = $\dfrac{n(n+1)}{2}$

Since, the torn page will have two page numbers.

Thus, the sum of all the page numbers including the torn page must be greater than 195

Thus, $\dfrac{n(n+1)}{2} > 195$

Or, $n(n + 1) > 390$ [Close to 390]

Here, the value of 'n' must be a natural number.

For 'n' = 19; $n(n + 1) = 19 \times 20 = 380 < 390$

For 'n' = 20; $n(n + 1) = 20 \times 21 = 420 > 390$

So, $n = 20$

Actual sum of all the page numbers = $\dfrac{20 \times 21}{2} = 210$

Sum when one of the pages is torn = 210

So, sum of the page numbers on the torn page

= 210 – 195 = 15

Let the page number on the torn page be 'x' and $(x + 1)$

So, $x + (x + 1) = 15$

Or, $2x = 14$

So, $x = 7$

Thus, the page number on torn page is '7' and '8'.

Hence, option (b) is correct.

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ARITHMETIC MEAN (AVERAGE)

If $x_1, x_2,.....x_n$ be n observations, then their arithmetic mean is given by

$$\overline{x} = \frac{x_1 + x_2 + ... + x_n}{n}$$

MEDIAN

To calculate the median of 'n' number of observations

- Arrange the observations in order, either from smallest to largest or largest to smallest.
- If 'n' is odd then, median is the middle value.

  i.e., Median (M) = Value of $\left(\dfrac{n+1}{2}\right)th$ observation

- If 'n' is even then, the median is the average of the two middle values.

  i.e., Median (M)

$$= \frac{\text{Value of } \left(\dfrac{n}{2}\right)\text{th observation} + \text{value of } \left(\dfrac{n}{2}+1\right)\text{th observation}}{2}$$

MODE

This is the number that appears most frequently in a group of numbers. It's like the most popular or common value in the set.

For example: Consider the daily sales of a store:
12, 15, 18, 15, 20, 15, 18, 12, 15, 18.

Frequencies: 12 appears 2 times, 15 appears 4 times, 18 appears 3 times, and 20 appears 1 time.

The most frequent value is 15, occurring 4 times.

Mode: 15, as it represents the most common daily sales value.

RELATION BETWEEN MEAN, MEDIAN AND MODE

The following empirical relationship exists between the Mean, Median and Mode:

$$\textbf{Mode} = 3 \times \textbf{Median} - 2 \times \textbf{Mean}$$

1. There are thirteen 2-digit consecutive odd numbers. If 39 is the mean of the first five such numbers, then what is the mean of all the thirteen numbers?

(UPSC CSAT 2017)

(a) 47   (b) 49
(c) 51   (d) 45

Sol: Sum of the five consecutive odd numbers
= 39 × 5 = 195

Let the five consecutive odd numbers be $(x - 4)$, $(x - 2)$, 'x', $(x + 2)$, $(x + 4)$

According to the question;

$(x - 4) + (x - 2) + x + (x + 2) + (x + 4) = 195$

Or, $5x = 195$

So, $x = 39$

So, the numbers are: 35, 37, 39, 41, 43, …………..

This follows an *AP* with first term (a) = 35 and common difference (d) = 2

$$\text{Sum of 13 terms} = \frac{13}{2}\Big[2\times35 + (13-1)\times2\Big]$$

Mean of these 13 terms

$$= \frac{\textit{sum of the 13 terms}}{13} = \frac{1}{13}\times\frac{13}{2}\Big[2\times35+(13-1)\times2\Big] = \frac{1}{2}\Big[70+24\Big] = 47$$

Hence, option (a) is correct.

2. If for a sample data, Mean < Median < Mode, then the distribution is **(UPSC CSAT 2017)**

(a) symmetric
(b) skewed to the right
(c) neither symmetric nor skewed
(d) skewed to the left

Sol: Arithmetic mean is also known as average. It is the sum of all the numbers divided by the number of numbers. Median is the middle value in the list of numbers written in ascending or descending order. Mode is the most frequently occurring value in the data set (*i*t corresponds to the peak of the bell shaped distribution curve).

A Normal Distribution is perfectly symmetrical (not skewed) and the mean is exactly at the peak of the bell shaped curve as shown below:

Symmetrical Distribution
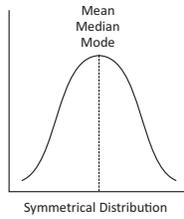
A distribution is skewed if one tail is longer than another. These distributions are sometimes called asymmetric distributions. These are of two types:

1. A right-skewed distribution has a long right tail (also called positively-skewed distribution). The mean is to the right of the peak/mode, i.e. Mean > Mode.

Positively or Right skewed
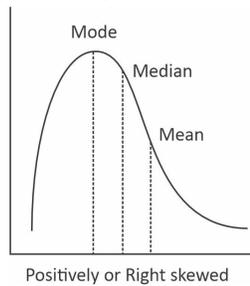
2. A left-skewed distribution has a long left tail (also called negatively-skewed distribution). The mean is to the left of the peak, i.e. Mean < Mode.

Negatively or Left skewed
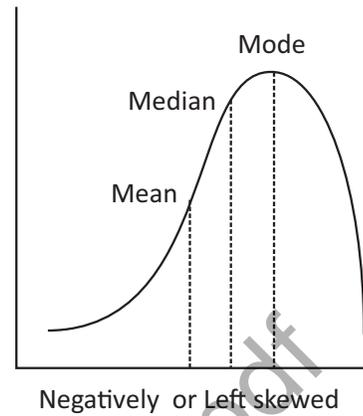
We can see in the above graph that Mean < Median and Mode, the condition given for the sample data in the question. Hence, the distribution of the sample data is skewed to the left.

Hence, option (d) is correct.

❖   ❖   ❖

KEY POINT

A pie chart either represent the distribution 100% of the data in terms of percentage or 360° of the data in terms of degrees.

So, 100% = 360°

So, 1% = 3.6°

And, $1° = \left(\dfrac{100}{360}\right)\% = \left(\dfrac{5}{18}\right)\%$

So, $x\% = x \times 3.6°$ and, $x° = \left(\dfrac{5}{18}\right) \times x\%$

1. **A pie chart gives the expenditure on five different items 'A', 'B', 'C', 'D' and 'E' in a household. If 'B', 'C', 'D' and 'E' correspond to 90°, 50°, 45° and 75°, respectively, then what is the percentage of expenditure on item 'A'?** **(UPSC CSAT 2022)**

(a) $\dfrac{112}{9}$　　　　　(b) $\dfrac{125}{6}$

(c) $\dfrac{155}{9}$　　　　　(d) $\dfrac{250}{9}$

Sol: The angle corresponding to expenditure on item 'A' = = 360° – Sum of angles corresponding to expenditure on 'B', 'C', 'D' and 'E' 360° – (90° + 50° + 45° + 75°) = *100°*

We know that;

For pie chart 360° = 100%

$1° = \left(\dfrac{100\%}{360°}\right)$

So, 100 degree $= \left(\dfrac{100\%}{360°}\right) \times 100° = \left(\dfrac{250}{9}\right)\%$

Thus, the percentage of expenditure on item

'A' $= \left(\dfrac{250}{9}\right)\%$

Hence, option (d) is correct.

Question Based on Bar Graph

Direction: The graph given below shows imports of three different types of steel over a period of six months of a year.

Study the graph and answer the question that follows.
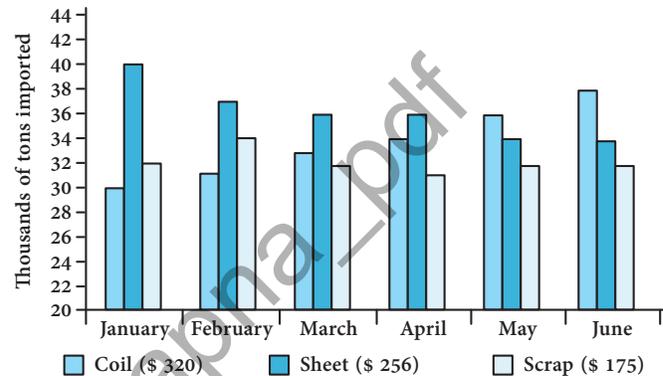
The figures in the brackets indicate the average cost per ton over a six month period.

2. **By how much (measured in thousands of tons) did the import of sheet steel exceed the import of coil steel in the first three months of the year?** **(UPSC CSAT 2018)**

(a) 11　　　　　(b) 15

(c) 19　　　　　(d) 23

Sol: Import of sheet steel in the first three months of the year
= 40 + 37 + 36 = 113 thousand tons

Import of coil steel in the first three months of the year
= 30 + 33 + 31 = 94 thousand tons

Since, 113 – 94 = 19

Thus, the import of sheet steel exceeded the import of coil steel by 19 thousand tons in the first three months of the year.

Hence, option (c) is correct.

Question Based on Line Graph

3. **Average hourly earnings per year (E) of the workers in a firm are represented in figures 'A' and 'B' as follows:**
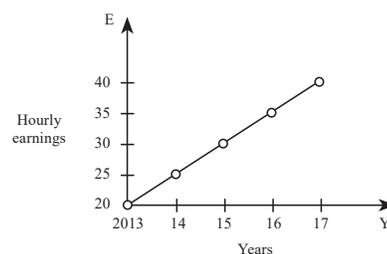
## Fig. A
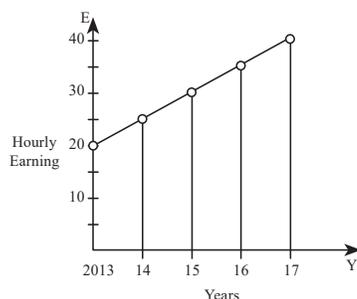
Fig. B

From the figures it is observed that

(*a*) values of '*E*' are different.

(*b*) ranges (*i.e.*, the difference between the maximum and the minimum) of '*E*' are different.

(*c*) slopes of the graphs are same.

(*d*) rates of increase of '*E*' are different.

Sol: **For option (*a*):** Since, the value of '*E*' for 15 years is same in figure '*A*' as well as '*B*'. Thus, option (*a*) is incorrect.

For option (*b*):

Range of '*E*' for figure '*A*' = 40 – 20 = 20

Range of '*E*' for figure '*B*' = 40 – 20 = 20

Thus, option (*b*) is incorrect.

For option (*c*):

We can observe tha in both the figures points are same *i.e.*, (2013, 20), (2014, 25), (2015, 30), (2016, 35) and (2017, 40)

So if graphs are same then slops are same or Slope of a straight line

$$\text{Slope} = \frac{\text{Value at } b - \text{Value at } a}{b - a}$$

For figure '*A*':

a = 2016 and b = 2015

$$\text{Slope of figure '}A\text{'} = \frac{35 - 30}{2016 - 2015} = 5$$

For figure '*B*':

a = 2015 and b = 2017

$$\text{Slope of figure '}B\text{'} = \frac{40 - 30}{17 - 15} = 5$$

Thus, the slopes of both the graphs are the same.
Hence, option (c) is correct.

Question based on Table

4. **Consider the following Table:**

Player	Runs Scored in the First innings	Balls faced in the First Innings	Runs Scored in the Second innings	Balls faced in the Second Innings
A	61	99	14	76
B	05	12	50	85
C	15	75	20	50
D	13	55	12	50

Who is the fastest run scorer in the Test Match?

(*a*) *A* (*b*) *B*

(*c*) *C* (*d*) *D*

Sol: The fastest run scorer will be the person who scores more runs in less balls i.e the person for whom ratio of runs scored to balls faced is maximum

For person '*A*':

$$\text{Runs scored: Balls faced} = \frac{75}{175} \sim 0.43$$

For person '*B*':

$$\text{Runs scored: Balls faced} = \frac{55}{97} \sim 0.57$$

For person '*C*':

$$\text{Runs scored: Balls faced} = \frac{35}{125} \sim 0.28$$

For person '*D*':

$$\text{Runs scored: Balls faced} = \frac{25}{105} = 0.24$$

The best ratio is that of person '*B*'.

Hence, option (b) is correct.

❖   ❖   ❖

KEY CONCEPTS

- **Pythagoras Theorem:** It states that in a right angled triangle, the square of the length of the hypotenuse (the side opposite the right angle) is equal to the sum of the squares of the lengths of the other two sides.
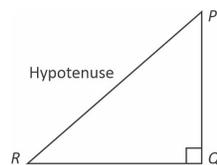

In ΔPQR, $\angle Q = 90°$

So, $PR^2 = PQ^2 + RQ^2$

1. **In a plane, line 'X' is perpendicular to line 'Y' and parallel to line 'Z'; line 'U' is perpendicular to both lines 'V' and 'W'; line 'X' is perpendicular to line 'V'. Which one of the following statements is correct?**

   (UPSC CSAT 2015)

   (*a*) 'Z', 'U' and 'W' are parallel.
   (*b*) 'X', 'V' and 'Y' are parallel.
   (*c*) 'Z', 'V' and 'U' are all perpendicular to 'W'.
   (*d*) 'Y', 'V' and 'W' are parallel.

**Sol:** We can make the diagram below based on the information given
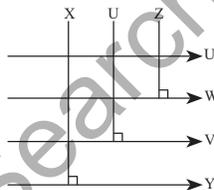
We can see that lines 'Y', 'V' and 'W' are parallel to each other

Hence, option (d) is correct.

2. *AB* **is a vertical trunk of a huge tree with 'A' being the point where the base of the trunk touches the ground. Due to a cyclone, the trunk has been broken at 'C' which is at a height of 12 meters, broken part is partially attached to the vertical portion of the trunk at 'C'. If the end of the broken part 'B' touches the ground at 'D' which is at a distance of 5 meters from 'A', then the original height of the trunk is: (UPSC CSAT 2016)**

   (*a*) 20 m   (*b*) 25 m
   (*c*) 30 m   (*d*) 35 m

Sol: Given scenario can be describe using the following figure:
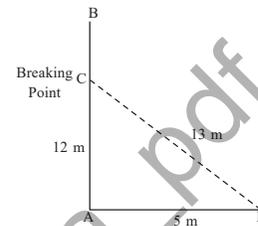
Here, $CD = CB$

In right triangle CAD

Using pythagoras theorem, we have;

$AD^2 + AC^2 = CD^2$

Or, $CD^2 = AD^2 + AC^2 = 5^2 + 12^2 = 25 + 144 = 169$

So, $CD = 13$ m

So, original height of the trunk $= AC + CB = AC + CD$
$= 12 + 13 = 25$ m

Hence, option (b) is correct.

3. **Consider the following statements:**

   I. The minimum number of points of intersection of a square and a circle is 2.

   II. The maximum number of points of intersection of a square and a circle is 8.

   Which of the above statements is/are correct?

   (UPSC CSAT 2020)

   (*a*) I only   (*b*) II only
   (*c*) Both I and II   (*d*) Neither I nor II

Sol: **For statement 'I':**

Minimum Number of Points of Intersection:
A circle and a square can have zero points of intersection if the circle is entirely outside the square or entirely inside without touching the square. Therefore, the minimum number of points of intersection between a square and a circle is 0, not 2.

Thus, statement I is incorrect.

For statement 'II':

The maximum number of points of intersection occurs when the circle intersects each side of the square at two points.

Since a square has 4 sides, the maximum number of intersection points is $4 \times 2 = 8$

Thus, statement II is correct.

Hence, option (b) is correct.
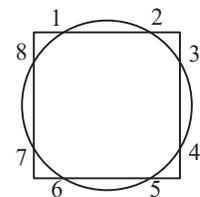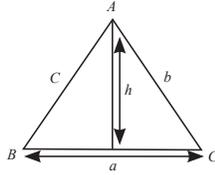
Triangle

Formulae given below are valid for all types of triangles.

- Perimeter = $(a + b + c)$
- Area = $\dfrac{1}{2} \times$ (base $\times$ height) = $\dfrac{1}{2} \times a \times h$

Rectangle
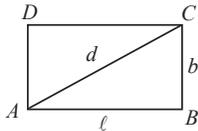
- Perimeter $(P) = 2 \times$ (length + breadth) = $2 \times (\ell + b)$
- Area $(A) = $ length $\times$ breadth $= \ell \times b$
- Length of Diagonal $= \sqrt{\ell^2 + b^2}$
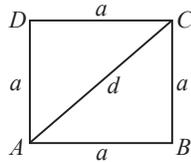
Square

- Perimeter $(P) = 4 \times$ (side) $= 4a$
- Area (A) of a square $= $ (side)$^2 = a^2$
- Length of diagonal $= \sqrt{2} \times Side = a\sqrt{2}$

Circles

If r is the radius of a circle, then

- Perimeter or Circumference $= 2\pi r$ or πd, where $d = 2r$ is the diameter of the circle.
- Area $= \pi r^2$ or $\dfrac{\pi d^2}{4}$
- Area of semi-circle $= \dfrac{\pi r^2}{2}$
- Area of a quadrant of a circle $= \dfrac{\pi r^2}{4}$

SURFACE AREA AND VOLUME FOR DIFFERENT SHAPES

Cuboid
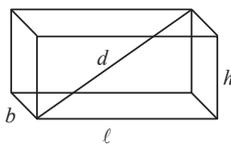
- $d = \sqrt{\ell^2 \times b^2 \times h^2}$
- Volume $= \ell \times b \times h$
- Total Surface Area $= 2(\ell b + bh + \ell h) = (l + b + h)^2 - d^2$
- Area of four walls of a room = Lateral Surface area
  $= 2(\ell + b)h$

Cube
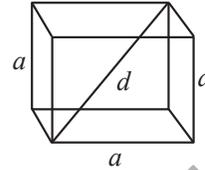
If 'a' be the edge of a cube, then

- Body diagonal of the cube, $d = \sqrt{3}a$
- Volume of the cube $= $ (edge)$^3 = a^3$
- Total surface area of the cube $= 6$ (edge)$^2 = 6a^2$
- Lateral Surface area $= 4$ (edge)$^2 = 4a^2$

Right Circular Cylinder

- Volume of cylinder = Area of the base $\times$ height
  $= \pi r^2 \times h = \pi r^2 h$
- Curved surface area = Circumference of the base $\times$ height
  $= 2\pi r \times h = 2\pi rh$
- Total surface area = Area of the curved surface + Area of the two circular ends
  $= 2\pi rh + 2\pi r^2 = 2\pi r(h + r)$

Hollow Right Circular Cylinder

- Outer curved surface area $= 2\pi Rh$
- Inner curved surface area $= 2\pi rh$
- Total curved surface area = Outer curved surface area + Inner curved surface area
  $= 2\pi Rh + 2\pi rh = 2\pi h(R + r)$
- Area of upper and lower cross-section
  $= \pi R^2 - \pi r^2 = \pi(R^2 - r^2)$
- Total surface area $= 2\pi h(R + r) + 2\pi(R^2 - r^2)$
- Volume $= \pi R^2 h - \pi r^2 h = \pi(R^2 - r^2)h$

Right Circular Cone

- $\ell = $ slant height $= \sqrt{h^2 + r^2}$
- Volume of cone $= \dfrac{\pi r^2 h}{3}$
- Area of curved surface $= \pi r\ell$
- Total surface area of cone = Area of the base + area of the curved surface $= \pi r^2 + \pi r\ell = \pi r(r + \ell)$

Sphere

- Volume of sphere $= \dfrac{4}{3}\pi r^3$
- Surface area $= 4\pi r^2$

Hemisphere

- Volume of hemisphere $= \dfrac{2}{3}\pi r^3$
- Curved surface Area $= 2\pi r^2$
- Total surface area $= 3\pi r^2$

Now Let's solve some questions based on the above concepts

1. **Consider the following statements in respect of a rectangular sheet of length 20 cm and breadth 8 cm:**

   I. It is possible to cut the sheet exactly into 4 square sheets.

   II. It is possible to cut the sheet into 10 triangular sheets of equal area.

   Which of the above statements is/are correct?

   (UPSC CSAT 2022)

   (a) I only   (b) II only

   (c) Both I and II   (d) Neither I nor II

Sol: It is given that the rectangle is of dimensions 20 cm × 8 cm.

Statement I: We need to cut the given rectangle into 4 squares (of any area, not identical)
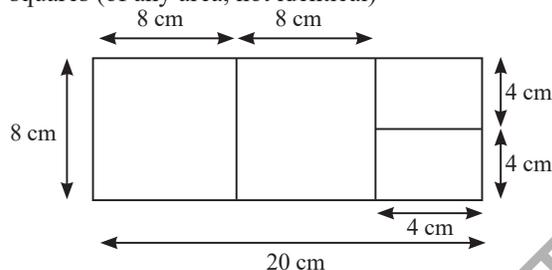
Here area of 2 squares = 8 × 8 = 64 cm²

And, area of other two squares = 4 × 4 = 16 cm²

Statement II: The given rectangle can be cut into 10 triangles of equal area as shown below.
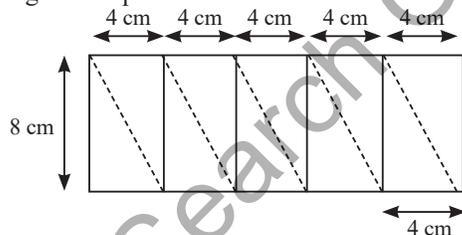
Here, each traingle is a right angled taingle and area of each triangle = (1/2) × 8 × 4 = 16 cm²

So, both statements I and II are correct.

Hence, option (c) is correct.

2. **An agricultural field is in the form of a rectangle having length 'X_1' meters and breadth 'X_2' meters ('X_1' and 'X_2' are variable). If $(X_1 + X_2) = 40$ meters, then the area of the agricultural field will not exceed which one of the following values?**

   (UPSC CSAT 2016)

   (a) 400 sq m   (b) 300 sq m

   (c) 200 sq m   (d) 80 sq m

Sol: Given, $X_1 + X_2 = 40$

Area of rectangular field $= X_1 \times X_2$

(We know that, of all the rectangles, a square has the largest area.)

Thus, for maximum area of the rectangle field, 'X_1' will be equal to 'X_2'

Given, $X_1 + X_2 = 40$

Or, $X_1 + X_1 = 40$

Or, $2X_1 = 40$

So, $X_1 = 20$

Thus, $X_1 = X_2 = 20$

Maximum area of rectangle

$= X_1 \times X_2 = 20 \times 20 = 400$ sq. m

Thus, the area of the agricultural field will not exceed 400 sq.m.

Hence, option (a) is correct.

3. **A piece of tin is in the form of a rectangle having length 12 cm and width 8 cm. This is used to construct a closed cube. The side of the cube is:**

   (UPSC CSAT 2016)

   (a) 2 cm   (b) 3 cm   (c) 4 cm   (d) 7 cm

Sol: Area of rectangle = length × width = 12 × 8 = 96 sq. cm

Let the side of the cube formed be 'a' cm

Surface area of cube = Area of rectangle

So, $6a^2 = 96$

Or, $a^2 = 16$

$\Rightarrow a = 4$

Thus, side of the cube = 4 cm

Hence, option (c) is correct.

4. **A village having a population of 4000 requires 150 liters of water per head per day. It has a tank measuring 20 m × 15 m × 6 m. The water of this tank will last for**   **(UPSC CSAT 2011)**

   (a) 2 days   (b) 3 days

   (c) 4 days   (d) 5 days

Sol: Population of the village = 4000

Water requirement per head per day = 150 l

Water requirement of the whole village per day = 4000 × 150 = 600000 l

Now it is given that village has a tank capacity of length = 20 m

Breadth = 15 m and height = 6 m

The volume of the tank = length × breadth × height = 20 m × 15 m × 6 m = 1800 m³

Now as we know that 1 m³ = 1000 l

Volume of the tank in litres = 1800 × 1000 = 1800000 l

Water consumed by the village in 1 day = 600000 l

Tank of the water will last for : $\dfrac{1800000}{600000} = 3$ days

Hence, option (b) is correct.

Data Sufficiency questions in quantitative aptitude test your ability to determine whether the provided information is adequate to answer a specific question. Typically, a question is followed by two statements, and your task is to assess if the statements individually or together provide enough data to answer the question.

Types of Options in Data Sufficiency Questions

In most cases, the options provided in these questions are as follows:

- Statement I alone is sufficient to answer the question, but Statement II alone is not sufficient.
- Statement II alone is sufficient to answer the question, but Statement I alone is not sufficient.
- Either Statement alone is sufficient to answer the question.
- Both Statements together are needed to answer the question, but neither Statement alone is sufficient.
- Neither Statement alone nor both together are sufficient to answer the question.

Structured Approach to Solve Data Sufficiency Problems:

To tackle these questions effectively, follow this step-by-step process:

Step 1: Understand the Question

Identify the Objective: Determine precisely what the question is asking. Focus on the nature of the problem (e.g., finding a value, proving a condition, or making a decision).

Recognize the Required Information: Understand the type of data needed to answer the question uniquely. For example, identifying a unique value or condition means the information must eliminate all ambiguities.

Step 2: Evaluate Each Statement Independently

Statement I:

Analyze if this statement alone provides sufficient information to answer the question.

If the information in Statement I leads to a single, definitive answer, it is sufficient.

If multiple answers are possible, it is not sufficient.

Statement II:

Similarly, assess if Statement II alone is sufficient to answer the question.

The same criterion applies, the statement must lead to a unique, unambiguous answer.

Step 3: Combine the Statements (If Necessary)

If neither Statement I nor Statement II is sufficient individually, analyze the data provided by both statements together.

Determine if combining the information allows you to answer the question definitively.

Key Points to Remember

The answer to the question must be unique. If a statement or combination of statements leads to multiple possible answers, the data is insufficient.

Do not assume additional information: Use only the data explicitly provided in the statements.

Work step-by-step: Evaluate each statement carefully before combining them.

Now lets try to apply this method to solve the questions:

1. **Consider a 3-digit number.**

   Question: What is the number?

   Statement-I: The sum of the digits of the number is equal to the product of the digits.

   Statement-II: The number is divisible by the sum of the digits of the number.

   Which one of the following is correct in respect of the above Question and the Statements?

   (UPSC CSAT 2023)

   (*a*) The Question can be answered by using one of the Statements alone, but cannot be answered using the other Statement alone.

   (*b*) The Question can be answered by using either Statement alone.

   (*c*) The Question can be answered by using both the Statements together, but cannot be answered using either Statement alone.

   (*d*) The Question cannot be answered even by using both the Statements together.

Sol: **Statement-I:** The sum of the digits of the number is equal to the product of the digits.

If the 3-digit number can be: 123, 132, 213, 231, 312, or 321, then the sum of the digits of the number would be equal to the product of the digits.

As, 1 + 2 + 3 = 1 × 2 × 3

Or, 6 = 6

Since, we cannot determine a unique value of the three digit number.

So, Statement-I alone is not sufficient to answer the question.

Statement-II: The number is divisible by the sum of the digits of the number.

If we consider two numbers i.e. 132 and 312, then

we can see that, $\dfrac{132}{6}$ = 22, and $\dfrac{312}{6}$ = 52

Since, we cannot determine a unique value of the three digit number.

So, Statement-II alone is not sufficient to answer the question.

After combining statements I and II together also, we will have 132 and 312 as the three digit numbers which satisfy both the conditions i.e. we cannot determine a unique 3 digit number even after combining both statements.

Thus, the Question cannot be answered even by using both the Statements together.

Hence, option (d) is correct.

2. **A Question is given followed by two Statements I and II. Consider the Question and the Statements.**

   Question: What are the values of m and n, where m and n are natural numbers?

   Statement-I: m + n > mn and m > n.

   Statement-II: The product of m and n is 24.

   Which one of the following is correct in respect of the above Question and the Statements?

   (*a*) The Question can be answered by using one of the Statements alone, but cannot be answered using the other Statement alone.

   (*b*) The Question can be answered by using either Statement alone.

   (*c*) The Question can be answered by using both the Statements together, but cannot be answered using either Statement alone.

   (*d*) The Question cannot be answered even by using both the Statements together.

Sol: **From statement-I:**

We are given that (m + n) > mn and m > n

Here, more than one value is possible as we can take n = 1, m = 2; n = 1, m = 3 etc.

So, statement-I alone is not sufficient to answer the question.

From statement-II:

m × n = 24

Here, m and n can be (1, 24), (2, 12), (3, 8), (4, 6), (6, 4), (8, 3), (12, 2) and (24, 1)

So, statement II alone is not sufficient to answer the question.

Combining statements I and II, we get;

Only one value will satisfy the conditions given in both statements i.e. m = 24 and n = 1

Therefore, both statements together are required to answer the question.

Hence, option (c) is correct.

3. **A Question is given followed by two Statements I and II. Consider the Question and the Statements.**

   Age of each of P and Q is less than 100 years but more than 10 years. If you interchange the digits of the age of P, the number represents the age of Q.

   Question: What is the difference of their ages?

   Statement-I: The age of P is greater than the age of Q.

   Statement-II: The sum of their ages is 11/6 times their difference.

   Which one of the following is correct in respect of the above Question and the Statements? **(UPSC CSAT 2024)**

   (*a*) The Question can be answered by using one of the Statements alone, but cannot be answered using the other Statement alone.

   (*b*) The Question can be answered by using either Statement alone.

   (*c*) The Question can be answered by using both the Statements together, but cannot be answered using either Statement alone.

   (*d*) The Question cannot be answered even by using both the Statements together.

Sol: Let age of 'P' = ($10x$ + y) years

So, age of 'Q' = (10y + x) years

Statement-I:

10x + y > 10y + x

Or, 9x > 9y

Or, x > y

Since, we cannot find unique value of 'x' and 'y'.

So, this statement is not sufficient.

Statement-II:

Case '1':

Let age of 'P' > age of 'Q', then

(10x + y) + (10y + x) = $\dfrac{11}{6}$ ×[(10x + y) – (10y + x)]

Or, $11(x + y) = \dfrac{11}{6} \times (9x - 9y)$

Or, $6x + 6y = 9x - 9y$

Or, $3x = 15y$

Or, $x = 5y$

So, age of 'P' = $10 \times 5y + y$ = '51y' years

And, age of 'Q' = $10y + 5y$ = '15y' years

Since, age of 'P' and 'Q' is more than 10 and less than 100 years

So, 'y' must be 1

Thus, age of 'P' and 'Q' will be 51 years and 15 years, respectively

Required difference = $51 - 16 = 36$ years

Case '2':

Let age of 'P' < age of 'Q', then

$(10x + y) + (10y + x) = \dfrac{11}{6} \times [\,(10y + x) - (10x + y)]$

Or, $11(x + y) = \dfrac{11}{6} \times (9y - 9x)$

Or, $6x + 6y = 9y - 9x$

Or, $3y = 15x$

Or, $y = 5x$

So, age of 'Q' = $10 \times 5x + x$ = '51x' years

And, age of 'P' = $10x + 5x$ = '15x' years

Since, age of 'P' and 'Q' is more than 10 and less than 100 years

So, 'x' must be 1

Thus, age of 'P' and 'Q' will be 15 years and 51 years, respectively

Required difference = $51 - 16 = 36$ years

Since the difference in both cases is same.

Therefore, statement 2 alone is sufficient.

Thus, the Question can be answered by using one of the Statements alone, but cannot be answered using the other Statement alone.

Hence, option (a) is correct.

❖   ❖   ❖

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Logical Reasoning

1

Series

This is one of the most important chapter of logical reasoning for UPSC CSAT, the chapter covers three main types of series:

  I. Number series,   II. Alphabetical series

 III. Alphanumeric series

I. NUMBER SERIES

In this section, we deal with the questions in which a series of numbers (generally called the terms of the series) are given. These terms follow a certain pattern throughout the series. Basically, the following patterns can be asked in the exam:

1. **Series Based on Addition and Subtraction**

    Each number in the sequence is linked to the previous one through a simple rule of adding or subtracting a fixed amount. For example, if we start with a number and consistently add the same number to it, we get a series like 2, 5, 8, 11, where we're adding 3 each time.

2. **Series Based on Multiplication and Division**

    Each term in the sequence is connected to the previous one by either multiplying or dividing by a certain number. For instance, if we start with a number and continuously multiply it by the same number, like starting with 2 and multiplying by 3 each time, we get a series like 2, 6, 18, 54, and so on.

3. **Series Based on Prime Numbers**

    In a prime number series, the sequence involves operations like addition, subtraction, multiplication, or division using prime numbers. For example 4, 6, 10, 14, 22, 26. The term of the given series are two times the prime number (2, 3, 5, 7, 11, 13)

4. **Series Based on Addition/ Subtraction of Squares or Cubes of Numbers**

    In a series based on squares or cubes of numbers, we create each term by adding or subtracting the square or cube of a number, following a specific pattern. For example, if we start with a number and keep adding the square of another number (like 1, 2, 3, 4) to it, the series might look like 3 $(2+1^2)$, 7 $(3+2^2)$, 16 $(7+3^2)$, 30 $(14+4^2)$, and so on.

5. **Miscellaneous Series**

    Mixture/Miscellaneous series refers to a sequence of elements that follows a specific, but often complex, pattern. The challenge in these types of questions is to identify the underlying patterns that govern the series.

TYPE OF QUESTIONS

Type 1: To Find The Missing Term Of The Series.

Here we are given a series of numbers, including one or more missing terms. This series follows a certain pattern. By identifying that pattern, we need to find the missing term. With the help of the following question, we will understand the approach to solving this type of question:

1. **What is the value of X in the sequence:**
    2, 12, 36, 80, 150, X?   **(UPSC CSAT 2022)**

    (*a*) 248   (*b*) 252
    (*c*) 258   (*d*) 262

    Sol: The pattern has been represented below:
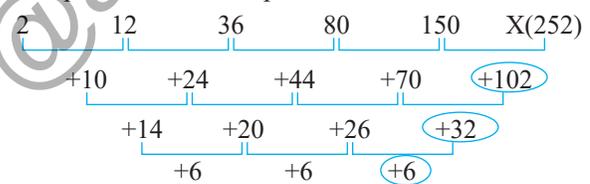
    This series is an example of Series based on addition and subtraction but we could not conclude any pattern in 1st differences. So, we checked the second differences (difference of differences) and we found a pattern as we can see in the above diagram.

    So the value of "X" is 252.

    Hence, option (b) is correct.

2. **What is the value of * in the given sequence:**
    3, 14, 39, 84, * , 258   **(UPSC CSAT 2024)**

    (*a*) 150   (*b*) 155
    (*c*) 160   (*d*) 176

    Sol:
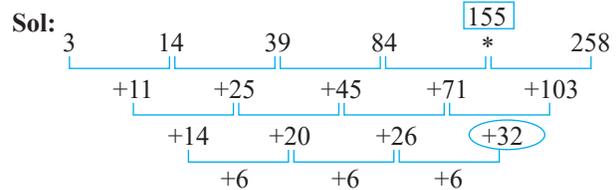
    Hence, option (b) is correct.

Type 2: To Find The Odd One Out

In this type of question, every term of a series except one term or one group follows a certain pattern and you will have to find that odd term or group. With the help of the following questions, we will understand the approach to solving this type of question:

1. **Choose the group which is different from the others:** (UPSC CSAT 2023)
   (*a*) 17, 37, 47, 97
   (*b*) 31, 41, 53, 67
   (*c*) 71, 73, 79, 83
   (*d*) 83, 89, 91, 97

Sol: We know that a prime number is a number that can only be divisible by either 1 or the number itself.

We can see that all the numbers in the said groups are prime numbers except 91 which is divisible by 13 as well as 7.

Hence, option (d) is correct.

2. **Replace the incorrect term with the correct term in the given sequence**

   3, 2, 7, 4, 13, 10, 21, 18, 31, 28, 43, 40

   Where odd terms and even terms follow the same pattern. (UPSC CSAT 2021)
   (*a*) 0
   (*b*) 1
   (*c*) 3
   (*d*) 6

Sol: In this series the terms at even position and at odd position follow different patterns:

The odd terms series is: 3, 7, 13, 21, 31, 43

And, the even terms series is: 2, 4, 10, 18, 28, 40

By separating odd and even series we can observe the following pattern

The Odd series pattern is:

$3 + 4 = 7$, $\quad 7 + 6 = 13$, $\quad 13 + 8 = 21$

$21 + 10 = 31$, $\quad 31 + 12 = 43$

So, the Even Series Pattern is:

$0 + 4 = 4$, $\quad 4 + 6 = 10$, $\quad 10 + 8 = 18$

$18 + 10 = 28$, $\quad 28 + 12 = 40$

So, '0' should come in place of '2' in the series.

Hence, option (a) is correct.

Type 3: Find The Missing Number In A Given Matrix

Generally, in these types of questions rows or columns follow a pattern. Our task is to identify the pattern and figure out the missing number in the matrix. With the help of the following question, we will understand the approach to solving this type of question:

1. **You are given two identical sequences in two rows:**

Sequence-I	8	4	6	15	52.5	236.25
Sequence-II	5	A	B	C	D	

   What is the entry in the place of 'C' for the Sequence II? (UPSC CSAT 2021)
   (*a*) 2.5
   (*b*) 5
   (*c*) 9.375
   (*d*) 32.8125

Sol: The pattern in the two sequences is as given below:

Sequence-I follow the pattern as:

$8 \times 0.5 = 4$, $\quad 4 \times 1.5 = 6$, $\quad 6 \times 2.5 = 15$

$15 \times 3.5 = 52.5$, $\quad 52.5 \times 4.5 = 236.25$

Sequence-II follow the pattern as:

$5 \times 0.5 = 2.5 = A$, $\quad 2.5 \times 1.5 = 3.75 = B$

$3.75 \times 2.5 = 9.375 = C$

The entry in the place of 'C' for the Sequence-II will be $= 9.375$

Hence, option (c) is correct.

2. **Consider the table given below in which the numbers bear certain relationships among themselves along the row**

29	13	18
33	x	19
30	27	3

   Which one of the following numbers is the missing number indicated above by X? (UPSC CSAT 2014)
   (*a*) 19
   (*b*) 15
   (*c*) 14
   (*d*) 8

Sol: In the given matrix the sum of each row is 60, so rows of this matrix follow a pattern.

Row 1: $29 + 13 + 18 = 60$

Row 3: $30 + 27 + 3 = 60$

Row 2: $33 + X + 19 = 60$

$X = 8$

Therefore, the value of X is 8.

Hence, option (d) is correct.

Type 4: Miscellaneous Pattern Questions

In these types of questions the pattern or demand of the question will be different from the types and patterns we discussed earlier in this chapter, these can be the puzzles or series of dates, puzzle on numbers etc. We will try to understand these by solving some questions that UPSC asked in the previous years:

1. **Consider the sequence given below:**

   4/12/95, 1/1/96, 29/1/96, 26/2/96 …

   What is the next term of the series? (UPSC CSAT 2018)
   (*a*) 24/3/96
   (*b*) 25/3/96
   (*c*) 26/3/96
   (*d*) 27/3/96

Sol: In the given sequence:

2nd term $= 4/12/95 + 28$ days $= 1/1/96$

3rd term $= 1/1/96 + 28$ days $= 29/1/96$

4th term $= 29/1/96 + 28$ days $= 26/2/96$

The year 1996 is a leap year.

So, there are 29 days in February 1996.

So, 5th term will be $26/2/96 + 28$ days $= 25/3/96$

Hence, option (b) is correct.

2. **Consider the following sequence of numbers:**

   5 1 4 7 3 9 8 5 7 2 6 3 1 5 8 6 3 8 5 2 2 4 3 4 9 6

   How many odd numbers are followed by the odd number in the above sequence? (UPSC CSAT 2020)
   (*a*) 5
   (*b*) 6
   (*c*) 7
   (*d*) 8

Sol: Given sequence:

5 1 4 **7 3** 9 8 **5** 7 2 6 **3 1** 5 8 6 3 8 5 2 2 4 3 4 9 6

The odd numbers followed by another odd number are: 5, 7, 3, 5, 3, and 1 (as shown by bold and underlined numbers)

So, required number of odd numbers = 6

Hence, option (b) is correct.

II. ALPHABETICAL SERIES

These types of series are formed with the help of the English alphabet. These sequences can be created using different rules or patterns, such as:

1. **Skipping Letters:** This series is formed by skipping a set number of letters between each element in the sequence. For example, we can get a series like A, C, E, G, I, and so on, when B, D, F, H are skipped.

2. **Pair of Opposite Letters:** This involves pairing letters that are on opposite ends of the alphabet. For example, A is paired with Z, B with Y, C with X, and so on.

A	B	C	D	E	F	G	H	I	J	K	L	M
Z	Y	X	W	V	U	T	S	R	Q	P	O	N

3. **Continuous Sequence of Letters:** This is the simplest type, where you just use the letters of the alphabet in their regular order, like A, B, C, D, E, all the way up to Z. There's no skipping or jumping; each letter follows the next in the usual way.

Now let us look at the different types of questions an alphabetical series:

TYPE OF QUESTIONS

Type 1: Find The Missing Term of a Letter Series

These series are created by skipping a specific number of letters between each term. To solve these types of questions, you need to figure out the rule for how many letters are being skipped each time. Now lets try and understand this with the help of examples:

1. **Find the next term of the series: B, E, H, K, ?**

   (a) K (b) L
   (c) M (d) N

Sol: B , E , H , K , N
   with +3 between each

   The next letter is "N".
   Hence, option (d) is correct.

2. **Find the missing term in the following series: MQCP, NPEN, OOGL, PNIJ, ?**

   (a) QMKH (b) QKKH
   (c) QMJK (d) QMKI

Sol: Consider the 1st letter of each term as shown in figure:

   MQCP , NPEN , OOGL , PNIJ , Q....
   with +1 between each

   Hence, 1st letter of missing term will be P +1 = Q

   Now, 2nd letter of each term is as shown in figure:

   MQCP , NPEN , OOGL , PNIJ , QM....
   with −1 between each

   Hence the 2nd letter of the missing term will be N − 1 = M.
   (option (b) eliminated)

Now, 3rd letter of each term is as shown in figure:

MQCP , NPEN , OOGL , PNIJ , QMK....
   with +2 between each

Hence the 3rd letter of missing term will be I + 2 = K (option c eliminated)

Now, consider the 4th letter of the each term as shown in figure:

MQCP , NPEN , OOGL , PNIJ , QMKH
   with −2 between each

4th letter of 4th term = L − 2 = J

Hence, 4th letter of of missing term = J − 2 = H

So, the missing term is QMKH

Hence, option (a) is correct.

Type 2. Find The Missing Terms In Continuous Series

In continuous series, firstly count the total given letters and try to divide the series into equal parts to find the possible patterns in the given series. For example, a series with 15 letters can be divided into smaller parts of 3 or 5 letters to find the pattern in the series. The following Questions will help us in developing a better understanding:

1. **In the series: _b_a_ba_b_abab_aab; fill in the six blanks (_) using one of the following given four choices such that the series follows a specific order.**
   (UPSC CSAT 2021)

   (a) bababa (b) baabba
   (c) bbaabb (d) ababab

Sol: The given series is _b_a_ba_b_abab_aab.
   Total number of letters in this series = 18

   Now, we will break the series in 3 letters:
   _b_ / a_b / a_b / _ab / ab_ / aab

   By trying options, we can find out the pattern being followed in this series
   i.e. abb / aab

   Using this pattern, fill the blanks:
   a<u>bb</u> / a<u>a</u>b / a<u>b</u>b / a<u>a</u>b / ab<u>b</u> / aab

   Hence, option (d) is correct.

2. **Consider the sequence ABC__ABC_DABBCD_ABCD that follows a certain pattern. Which of the following complete the sequence?(UPSC CSAT 2023)**

   (a) DACB (b) CDAB
   (c) DCCA (d) DDCA

Sol: The given sequence: ABC_ _ABC_DABBCD_ABCD
   There are 20 elements in this series.
   We can break this sequence in four sets of 5 elements each.
   ABC_ _ /ABC_D/ABBCD/_ABCD
   The third set ABBCD gives us an idea that one letter is repeating itself.

   Now, we can check from the options to further confirm our observation.

Using option (d), we get the complete sequence: ABCDD/ABCCD/ABBCD/AABCD

We can see that in the first set the 4th letter is repeating, in the second set the 3rd letter is repeating, and so on.

Hence, option (d) is correct.

Type 3: Miscellaneous Pattern Questions

In this type, we will explore some different types of alphabetical series through Questions.

1. **What is the middle terms of the sequence Z, Z, Y, Y, Y, X, X, X, X, W, W, W, W, W,……,A?**
   (UPSC CSAT 2023)

   (*a*) H   (*b*) I
   (*c*) J   (*d*) M

Sol: The given sequence is: Z, Z, Y, Y, Y, X, X, X, X, W, W, W, W, W,..., A

So, the number of terms in the given sequence

= 2 + 3 + 4 + … + 27 +1 -1 = Sum of 1st 27 natural numbers – 1

$$= \left[\frac{27(27+1)}{2}\right] - 1 = 378 - 1 = 377$$

So the middle term = $\dfrac{Sum+1}{2} = \dfrac{377+1}{2} = \dfrac{378}{2}$ = 189th.

Now, 377 – (27(A) + 26(B) + 25(C) + 24(D) + 23(E) + 22(F) + 21(G) + 20(H)) = 189

So, the 189th term must be I.

Hence, option (b) is correct.

2. **In the series AABABCABCDABCDE.., which letter appears at the 100th place?**   **(UPSC CSAT 2022)**

   (*a*) G   (*b*) H
   (*c*) I   (*d*) J

Sol: Given series: AABABCABCDABCDE…..
Pattern followed: A, AB, ABC, ABCD, …….

The first-term has 1 letter, second-term has 2 letters, third-term has 3 letters and so on.

[Sum of first *n* natural numbers = $\dfrac{n(n+1)}{2}$]

Sum needs to be close to 100

For n = 13; sum = $\dfrac{13(13+1)}{2} = 91$

So, upto 13th term we will have 91 letters

Now 14th term will have 14 letters (which will contain the 100th letter) as:

92nd term	93rd term	94th term	95th term	96th term	97th term	98th term	99th term	100th term
A	B	C	D	E	F	G	H	I

Thus, the 100th letter is I.

Hence, option (c) is correct.

III. ALPHANUMERIC SERIES

This type of series is a combination of alphabetical series and numerical series and can follow any of the patterns we have discussed earlier in this chapter.

We need to keep in mind the following alphabetical positions in forward and reverse order:

A	B	C	D	E	F	G	H	I	J	K	L	M
1	2	3	4	5	6	7	8	9	10	11	12	13
N	O	P	Q	R	S	T	U	V	W	X	Y	Z
14	15	16	17	18	19	20	21	22	23	24	25	26

A	B	C	D	E	F	G	H	I	J	K	L	M
26	25	24	23	22	21	20	19	18	17	16	15	14
N	O	P	Q	R	S	T	U	V	W	X	Y	Z
13	12	11	10	g	8	7	6	5	4	3	2	1

We will try and understand these concepts through examples:

1. **What is the missing term of the series 9, P, 25, Q, 8, H, 16, ?, 18?**

   (*a*) A   (*b*) B
   (*c*) R   (*d*) I

Sol: The given series is 9, P, 25, Q, 8, H, 16, ?, 18
We observe that in the given series, each alphabet corresponds to the difference between numbers around that alphabet as shown in given table:

25 – 9 = 16	P
25 – 8 = 17	Q
16 – 8 = 8	H
18 – 16 = 2	B

So, B is the missing term in the series.

Hence, option (b) is correct.

2. **Following is a matrix of certain entries. The entries follow a certain trend row-wise. Choose the missing entry (?) accordingly.**

7B	10A	3C
3C	9B	6A
10A	13C	?

(UPSC CSAT 2021)

   (*a*) 9B   (*b*) 3A
   (*c*) 3B   (*d*) 3C

Sol: From the given table we need to identify the pattern for Alphabets and also for numbers.
Alphabet pattern: BAC, CBA, ACB (First two letters are shifting to the right of 3rd letter in the next row)
Number pattern:
10 – 7 = 3,
9 – 3 = 6
So, 13 – 10 = 3
So, the missing entry is 3B.

Hence, option (c) is correct.

❖   ❖   ❖

KEY CONCEPTS

Position of alphabets in forward order:

A	B	C	D	E	F	G	H	I	J	K	L	M
1	2	3	4	5	6	7	8	9	10	11	12	13

N	O	P	Q	R	S	T	U	V	W	X	Y	Z
14	15	16	17	18	19	20	21	22	23	24	25	26

Position of alphabets in backward order:

A	B	C	D	E	F	G	H	I	J	K	L	M
26	25	24	23	22	21	20	19	18	17	16	15	14

N	O	P	Q	R	S	T	U	V	W	X	Y	Z
13	12	11	10	9	8	7	6	5	4	3	2	1

- **Backward position of letter = 27 – forward position of letter.**
- **Opposite Letter Position = (26 – Forward Position of that letter) + 1**

Opposite Letters

A	B	C	D	E	F	G	H	I	J	K	L	M
Z	Y	X	W	V	U	T	S	R	Q	P	O	N

Tricks to remember the position of Alphabet:

Remembering the position of each alphabet can be quite challenging, so we often create simple tricks to help us recall their place values easily. Some of them are discussed below:

Trick 1: CFILORUX and EJOTY Formula:

C	F	I	L	O	R	U	X
3	6	9	12	15	16	18	21

Things will become easier if you remember and use these formulas. In the 1st formula, we have the position values of C, F, I, L, O, R, U, and X, which are the multiples of THREE, which will make it easier to remember the position value of the alphabet in the forward direction

E	J	O	T	Y
5	10	15	20	25

The position values of E, J, O, T and Y, which are the multiples of FIVE.

Trick 2: To find the position value in reverse order, we can use the 1st formula by reversing the alphabet as shown in the figure below:

X	U	R	O	L	I	F	C
3	6	9	12	15	18	21	24

TYPE OF QUESTIONS

Type 1: Letter Coding

This type of question involves words where letters are substituted with other letters based on a specific pattern or rule to create a code. Our task is to identify the coding pattern or rule and then answer the subsequent questions based on this code.

We will try to understand the method for solving these questions by working through some examples:

Here's a simple approach:

1. If 'ZERO' is written as 'CHUR', then how is 'PLAYER' written? **(UPSC CSAT 2023)**

   (a) SOCACT   (b) SODBGT
   (c) SODBHT   (d) SODBHU

Sol: For this we need to know the position of alphabets. Here, we are required to decode the pattern. ZERO is written as CHUR.

We can see that the underlying pattern is very simple, as

Z + 3 = C, E + 3 = H, R + 3 = U, O + 3 = R

So, we can analyse that every letter of 1st word is coded by adding 3 in that letter.

We will follow the same pattern to code PLAYER.

$P+3=S, L+3=O, A+3=D, Y+3=B, E+3=H, R+3=U$

So, the required code is SODBHU.

Hence, option (d) is correct.

2. If the order of the letters in the English alphabet is reversed and each letter represents the letter whose position it occupies, then which one of the following represents 'LUCKNOW'? **(UPSC CSAT 2022)**

   (a) OGXPMLD   (b) OGXQMLE
   (c) OFXPMLE   (d) OFXPMLD

Sol: In this question pattern is already given we just need to execute that pattern on given word:

So, we need to find the opposite letter to the letters given in the word LUCKNOW.

We know that: Opposite Letter Position = 27 – Letter Position

So, in the case of LUCKNOW:

The word associated to LUCKNOW is "OFXPMLD"

Hence, option (d) is correct.

3. The letters of the word "INCOMPREHENSIBILITIES" are arranged alphabetically in reverse order. How many positions of the letter/letters will remain unchanged? **(UPSC CSAT 2023)**

(*a*) None   (*b*) One

(*c*) Two   (*d*) Three

Sol: In this question also the pattern is given.

So, Let us arrange the letters of the word "INCOMPREHENSIBILITIES" in reverse order, the resultant is: TSSRPONNMLIIIIIHEEECB

Now, we can find the letters with unchanged positions.

I	N	C	O	M	P	R	E	H	E	N	S	I	B	I	L	I	T	I	E	S
T	S	S	R	P	O	N	N	M	L	I	I	I	I	I	H	E	E	E	C	B

INCOMPREHENSIBILITIES

So, the position of "I" is unchanged.

Hence, option (c) is correct.

Type 2: Number/Symbol Coding

In these types of questions, words are converted into numbers, or numbers are turned into letters using a specific pattern or rule. To solve them, figure out the rule used for the conversion.

1. If in a certain code, 'ABCD' is written as 24 and 'EFGH' is written as 1680, then how is 'IJKL' written in that code? **(UPSC CSAT 2024)**

(*a*) 11880   (*b*) 11240

(*c*) 7920   (*d*) 5940

Sol: Here we know the positions of alphabets i.e. alphabetical position: A = 1, B = 2, etc

Now understand the coding:

A	B	C	D	
1	2	3	4	→ 1 × 2 × 3 × 4 = 24
E	F	G	H	
5	6	7	8	→ 5 × 6 × 7 × 8 = 1680
I	J	K	L	
9	10	11	12	→ 9 × 10 × 11 × 12 = 11880

Hence, option (a) is correct.

2. If $7 \oplus 9 \oplus 10 = 8$, $9 \oplus 11 \oplus 30 = 5$, $11 \oplus 17 \oplus 21 = 13$, what is the value of $23 \oplus 4 \oplus 15$? **(UPSC CSAT 2023)**

(*a*) 6   (*b*) 8

(*c*) 13   (*d*) 15

Sol: From the above equations one pattern is observed.

Here, we are just adding up the numbers and then summing up the digits of the resultant number.

$\Rightarrow 7 + 9 + 10 = 26 \Rightarrow 2 + 6 = 8$

$\Rightarrow 9 + 11 + 30 = 50 \Rightarrow 5 + 0 = 5$

$\Rightarrow 11 + 17 + 21 = 49 \Rightarrow 4 + 9 = 13$

So, $23 \oplus 4 \oplus 15 = 23 + 4 + 15 = 42$

$\Rightarrow 4 + 2 = 6$

Hence, option (a) is correct.

Type 3: Deciphering Message Word Coding

To solve these questions, compare two messages with a common word or number. The shared code represents the common word or number. Repeat this process with other message pairs to decode all words or numbers.

Let's look at some examples to get a clearer understanding of how to approach these questions:

1. In a certain code, '256' means 'red colour chalk','589' means 'green colour flower' and '254' means 'white colour chalk'. The digit in the code that indicates 'white' is **(UPSC CSAT 2017)**

(*a*) 2   (*b*) 4

(*c*) 5   (*d*) 8

Sol: Now it is given that

'256' = red colour chalk   ...(*i*)

'589' = green colour flower   ...(*ii*)

'254' = white colour chalk   ...(*iii*)

In (*i*) and (*ii*) 'colour' and '5' is common

So, 5 = colour

In (*i*) and (*iii*) 'chalk' and '2' is common

So, 2 = chalk

From (*iii*), we have

5 = colour

2 = chalk

Therefore, '4' indicates white.

Hence, option (b) is correct.

2. In a certain code language, 'ma pa na' means 'I am male' and 'ma sa na' means ' I am female'. Which of the following means 'male'?

(*a*) ma   (*b*) pa

(*c*) na   (*d*) sa

Sol: In a given code,

ma pa na → I am male

ma sa na → I am female

Here, 'ma' and 'na' are common in both the codes and 'I' and 'am' are common in both the messages.

Hence, the code for male is 'pa'.

Hence, option (b) is correct.

Type 4: Miscellaneous Coding

Miscellaneous Coding in coding-decoding involves varied patterns that don't follow standard rules. It tests your ability to identify unique logic. Common types include:

1. **Substitution Codes:** Words or numbers are replaced with unrelated terms or symbols.

2. **Mathematical Operations:** Numbers or letters undergo specific operations (e.g., addition, reversal).

3. **Mixed Letter and Number Codes:** Combines letters and numbers based on position or operations.

Now let us understand it with the help of following questions:

1. **The local terminology for 'earth', 'tree', 'food', 'water' and 'star' on another planet is 'food', 'water', 'star', 'tree' and 'earth' respectively. If someone is hungry there, then what would he eat?**

   (a) star   (b) food

   (c) earth   (d) tree

Sol: **On another planet:** earth → food, tree → water, food → star, water → tree, stars → earth

We know that if a person is hungry, he eats food and food is called star there. So, he eats star.

Hence, option (a) is correct.

2. **In some code, letters P, Q, R, S, T represent numbers 4, 5, 10, 12, 15. It is not known which letter represents which number. If Q – S = 2S and T = R + S + 3, then what is the value of P + R – T?** **(UPSC CSAT 2024)**

   (a) 1   (b) 2

   (c) 3   (d) Cannot be determined due to insufficient data

Sol: Given the condition that: Q – S = 2S

We can deduce that: Q = 3S which implies Q is multiple of 3.

So the possible values for Q are 12 and 15 and respective values of S are 4 and 5.

So **Case I:** Q = 12 and S = 4.

We have T = R + S + 3

So, T = R + 4 + 3 or T = R + 7

So, T – R = 7

This case is not possible as the difference of the two among 5, 10 and 15 cannot be '7'

Case II: Q = 15 and S = 5

Since, T = R + S + 3

So, T = R + 5 + 3

Or, T = R + 8

Only 12 = 4 + 8

So, T = 12 and R = 4

And, P = 10

So, P + R – T = 10 + 4 – 12 = 2

Hence, option (b) is correct.

3. **If a + b means a – b; a – b means a × b; a × b means a ÷ b; a ÷ b means a + b, then what is the value of 10 + 30 – 100 × 50 ÷ 25? (Operations are to be replaced simultaneously)** **(UPSC CSAT 2024)**

   (a) 15   (b) 0

   (c) –15   (d) –25

Sol: In this question we need to use BODMAS rule but before that we need to decode the operations given in the questions;

According to question;

10 + 30 – 100 × 50 ÷ 25

= 10 – 30 × 100 ÷ 50 + 25 = 10 – 30 × 2 + 25

= 10 – 60 + 25 = –25

Hence, option (d) is correct.

The face of a clock is split into 12 parts for hours and 60 smaller parts for minutes. There are three hands (hour, minute and second hand) on a clock: the big hand, called the minute hand, tells us the minutes; the small hand, known as the hour hand, tells us the hour. This design helps us read the time easily.
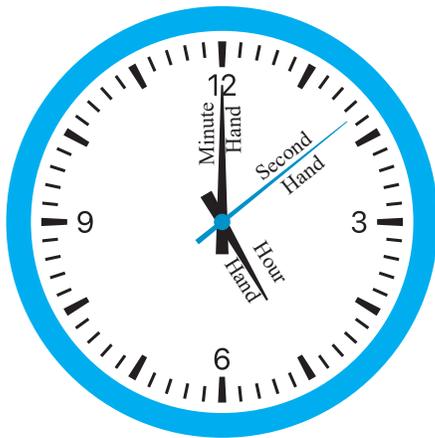
The clock represents three things i.e., **second, minute and hour.** The minute is a unit of time equal to $\frac{1}{60^{th}}$ of an hour or 60s i.e., **1 min = 60 sec**.

An hour is a unit of measurment for the time duration of 60 min or 3600 s.

i.e. **1 h = 60 min = 3600 sec**

Now we will discuss the different types of questions that are being asked from this chapter in the exam:

TYPE OF QUESTIONS

Type 1: Angle Between The Clock Hands

To solve clock angle problems, you need to understand how a clock's angles work:

- A clock is a circle, and a full circle is 360°. The clock has 12 numbers, so the distance between any two numbers (an hour space) is 30° because 360° ÷ 12 = 30°.
- The minute hand moves through 60 spaces in one full turn, which means it completes 360° in 60 minutes.
- It means, 60 min spaces = 360° ⇒ 1 min space = 6°.

When the hour hand completes a full circle, then it crosses 12 h spaces of the dial. It means, 12 h spaces = 360° implies 1 h space = 60 minutes = 30° ⇒ 1 min = 0.5°

Formula:

The angle between minute and hour hand = $\theta = \left| \frac{11M}{2} - 30H \right|$ where M = minute and H = hour.

Here in this formula we will use only the magnitude of $\left(\frac{11}{2M} - 30H \right)$ for calculating the angle.

Let us understand this through questions:

1. **What is the angle between the minute hand and hour hand when the clock shows 4:25 hours?**

   (*a*) 12.5°   (*b*) 15°
   (*c*) 17.5°   (*d*) 20°

Sol: **General method:**

At 4:25, the hour hand and minute hand are in specific positions:

The minute hand is pointing at the 25-minute mark.

The hour hand is slightly past the 4 because 25 minutes have passed since 4:00.

Position of minute hand:

We know that minute hand move 360° in 60 minutes or 6° in 1 minute.

So, in 25 minutes minute hand will move 150° (25 × 6) from 12.

Position of the Hour Hand:

The hour hand moves between numbers as time passes. At exactly 4:00, the hour hand is at the 4, which is: 4 × 30° = 120° from the 12.

But since 25 minutes have passed, the hour hand has moved further. The hour hand moves 30° per hour or 0.5° per minute. So, in 25 minutes, it moves: 25 × 0.5° = 12.5°.

Thus, the hour hand is now at: 120° + 12.5° = 132.5° from the 12.

Now the angle between the hands = 150° − 132.5° = 17.5°.

By Formula:

Here, M = 25 and H = 4

Required angle

$= \left| \frac{11}{2} \times M - 30 \times H \right| = \left| \frac{11}{2} \times 25 - 30 \times 4 \right| = 17.5°$

Hence, option (c) is correct.

2. **How many seconds in total are there in 'x' weeks, 'x' days, 'x' hours, 'x' minutes and 'x' seconds?**
(UPSC CSAT 2022)

   (*a*) 11580*x*   (*b*) 11581*x*

   (*c*) 694860*x*   (*d*) 694861*x*

Sol: We know that

    1 week = 7 Days

    1 day = 24 hours

    1 hour = 60 minutes

    1 minute = 60 seconds

    So, number of seconds in 'x' weeks

= 7 × 24 × 60 × 60 × *x* seconds = '604800*x*' seconds

So, number of seconds in 'x' days

= 24 × 60 × 60 × *x* seconds = '86400*x*' seconds

And, number of seconds in 'x' hours

= 60 × 60 × *x* seconds = '3600*x*' seconds

And, number of seconds in 'x' minutes

= 60 × *x* seconds = '60*x*' seconds

Therefore, total seconds in 'x' weeks, 'x' days, 'x' hours, 'x' minutes and 'x' seconds = 604800*x* + 86400*x* + 3600x + 60x + *x* = 694861x

Hence, option (d) is correct.

Type 2: The Positioning of Clock Hands

The general formula to find the time when the minute and hour hands are *t* minutes apart between *x* and *x* + 1 o'clock is given by:

$$\text{Time} = \frac{(5x \pm t) \times 12}{11} \text{ minutes past 'x'}$$

Here, *x* is the hour and *t* is the minute spacing between the hour and minute hand. **The '+' sign is used when the minute hand is ahead, and the '–' sign is used when the hour hand is ahead.**

Table: Clock Hand Positions and Their Frequencies

Details	Hands Forming Right Angles	Hands Aligning in a Straight Line (i) Hands Coincide	Hands Aligning in a Straight Line (ii) Opposite Positions
Condition	Right Angles	Coincide	Opposite Positions
Angle Between Hands	90°	0°	180°
Minute Spaces	15 minutes	0 minutes	30 minutes
Time Calculation Formula	$\frac{(5x \pm 15) \times 12}{11}$ minutes past x.	$\frac{60x}{11}$ minutes past x.	$\frac{(5x \pm 30) \times 12}{11}$ minutes past x.
Frequency in 12 Hours	22 times	11 times	11 times
Frequency in 24 Hours	44 times	22 times	22 times

Note: If both hands start moving together from the same position, they will conicide every $\frac{720}{11}$ minutes, approximately 65.45 minutes.

Let us solve some questions based on this concept:

1. **How many times the hour hand and the minute hand coincide in a clock between 10:00 a.m. and 2:00 p.m. (same day)?** (UPSC CSAT 2024)

   (*a*) 3 times   (*b*) 4 times

   (*c*) 5 times   (*d*) 6 times

Sol: Since, the minute and hour hand coincide 11 times in 12 hours i.e. one time in each hour while one time between 11:00 to 1:00 exactly at 12:00 noon. Thus, the minute and hour hand will coincide 3 times between 10:00 a.m. and 2:00 p.m.

10:00 am to 11:00 am ⇒ 1 time

11:00 am to 1:00 pm ⇒ 1 time

And, 1:00 pm to 2:00 pm ⇒ 1 time.

Hence, option (a) is correct.

2. **Between 6 PM and 7 PM the minute hand of a clock will be ahead of the hour hand by 3 minutes at** (UPSC CSAT 2015)

   (*a*) 6: 15 PM

   (*b*) 6: 18 PM

   (*c*) 6: 36 PM

   (*d*) 6: 48 PM

Sol: General Method:

At 6 PM hour hand is 180 degrees ahead of minute hand

We know that, Hour hand moves 0.5 degrees/minute Minute hand moves 6 degrees/minute Relative speed of minute hand w.r.t the hour hand = 6 – 0.5 = 5.5 degrees/minute.

We need minute hand to be 3 minutes ahead than hour hand i.e., 3 × 5.5 = 16.5 degrees

Minute hand will be 3 minutes ahead of hour hand at

$$= \frac{180 + 16.5}{5.5} = 35.7272 \text{ or } 6\text{: } 35.72 \text{ P.M.}$$

Thus, at 6:36 p.m., minute hand will be 3 minutes ahead of hour hand

By Formula:

We know the formula to find the time between "x" and "$x + 1$" when the minute hand is "t" minutes ahead of hour hand:

Time = $\dfrac{(5x \pm t) \times 12}{11}$ minutes past 'x'

And in this question $x = 6$ and $t = 3$

So, time $= \dfrac{(5 \times 6 + 3) \times 12}{11} = \dfrac{33 \times 12}{11} = 36$ minutes past 6.

Hence, option (c) is correct.

Type 3: Faulty Clocks

A faulty clock doesn't show the correct time.

- If it shows a time ahead of the actual time, it's "fast." For example, if the real time is 6:00, but the clock shows 6:10, it's 10 minutes fast.
- If it shows a time earlier than the actual time, it's "slow." For example, if the real time is 6:00, but the clock shows 5:50, it's 10 minutes slow.

Now let us understand the method to solve questions of this type:

1. **A wall clock moves 10 minutes fast in every 24 hours. The clock was set right to show the correct time at 8:00 a.m. on Monday. When the clock shows the time 6:00 p.m. on Wednesday, what is the correct time?** (UPSC CSAT 2019)

   (*a*) 5:36 p.m. (*b*) 5:30 p.m.

   (*c*) 5:24 p.m. (*d*) 5:18 p.m.

Sol: Since, the clock gains 10 minutes in 1 day or 24 hours

And, total hours between 8:00 am on Monday and 6:00 p.m. on Wednesday = 24 + 24 + 10 = 58 hours

So, minutes gained in 58 hours

$$= \frac{10}{24} \times 58 \sim 24 \text{ minurtes}$$

So, the correct time is = 6:00 p.m. – 24 minutes

= 5 : 36 p.m

Hence, option (a) is correct.

2. **A man started from home at 14:30 hours and drove to the village, arriving there when the village clock indicated 15:15 hours. After staying for 25 minutes, he drove back by a different route of length 1.25 times the first route at a rate twice as fast, reaching home at 16:00 hours. As compared to the clock at home, the village clock is** (UPSC CSAT 2022)

   (*a*) 10 minutes slow (*b*) 5 minutes slow

   (*c*) 10 minutes fast (*d*) 5 minutes fast

Sol: (*d*)

It is given that the total time taken by the man to come back home is 90 minutes, Out of which he stayed in the village for 25 minutes.

So, his total traveling time = 90 – 25 = 65 minutes

The return route was 1.25 times the initial route. So, time taken must have increased by 25% too. So, if the initial time taken was 't', now it must be 1.25t.

But it is also given that while returning he drove twice as fast. So, time taken must have been halved.

So, time taken while returning back $= \left(\dfrac{1.25t}{2}\right) = 0.625t$

Total time taken = t + 0.625t = 65 min

Or, 1.625t = 65 minutes

t = 40 minutes

So, the man took 40 minutes to reach the village.

So, the actual time at that moment = 14:30 + 40 minutes = 15:10 hours

It is clear that the village clock is 5 minutes fast.

Hence, option (d) is correct.

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A calendar is a system of organizing days. This is done by naming periods, typically days, weeks, months, and years. A date is a single and specific day within such a system.

Types of Year

- **Ordinary Year:** Has 365 days (52 weeks + 1 extra day). February has 28 days in an ordinary year.
- **Leap Year:** A leap year is an year which is divisible by 4. It has 366 days (52 weeks + 2 extra days). February has 29 days to align the calendar with Earth's ~365.25-day revolution.

  But a century year (divisible by 100) is a leap year when it is divisible by 400 i.e. 1600 is a leap year but 1700 is not a leap year.

Note: A leap year repeats itself every 28 years, and an ordinary year repeats itself in 6 or 11 years.

For Example, a person could use the same same calendar for the year 2000 (a leap year) and 2028.

ODD Days

Number of odd days in a given period = Remainder when Total number of days(between 2 given dates) is divided by 7.

Odd Days in Century

Table: Odd Days in Century

Year	Ordinary Years	Leap Years	Odd Days
100	76	24	5
200	152	48	3
300	228	72	1
400	303	97	0

Odd Days

Month	Number of Days	Odd Days
January	31 = 7 × 4 + 3	3
February	28 = 7 × 4 + 0 (Ordinary year)	0 (Ordinary year)
	29 = 7 × 4 + 1 (Leap year)	1 (Leap year)
March	31 = 7 × 4 + 3	3
April	30 = 7 × 4 + 2	2
May	31 = 7 × 4 + 3	3
June	30 = 7 × 4 + 2	2
July	31 = 7 × 4 + 3	3
August	31 = 7 × 4 + 3	3
September	30 = 7 × 4 + 2	2
October	31 = 7 × 4 + 3	3
November	30 = 7 × 4 + 2	2
December	31 = 7 × 4 + 3	3

TYPES OF QUESTIONS

Type 1: To find the day on a particular date when the reference date and day are given

METHOD:

Step 1: Calculate the number of days between the reference date and the target date. It involves counting the days in each month and accounting for leap years if necessary.

Step 2: Calculate the number of odd days between the reference date and the target date. This is done by dividing the total number of days between the dates by 7 and taking the remainder (odd days). Odd days represent the extra days beyond complete weeks.

Step 3: Move the given day of the reference date forward by the number of odd days we find in Step 2 to get the day on the target date.

Now let us understand this concept more clearly through some questions:

1. If the 3rd day of the month is Monday, which one of the following will be the fifth day from the 21st of this month? **(UPSC CSAT 2014)**

   (a) Monday   (b) Tuesday

   (c) Wednesday   (d) Friday

Sol: Given that the 3rd day of the month is Monday.

So 3 + 7 = 10 and 10 + 7 = 17 and 17 + 7 = 24th day is Monday.

So, 3, 10, 17 and 24 are Monday.

Now the 5th day from the 21st is the 26th.

Now the 24th is Monday and the 26th will be Wednesday.

Hence, option (c) is correct.

2. **Mr X has three children. The birthday of the first child falls on the 5th Monday of April, that of the second one falls on the 5th Thursday of November. On which day is the birthday of his third child, which falls on 20th December?** **(UPSC CSAT 2019)**

(a) Monday   (b) Thursday

(c) Saturday   (d) Sunday

Sol: As we know April has 30 days, 5th Monday of April is possible only on 29th April or 30th April

Case 1: When 29th April is 5th Monday

Total number of days from (30th April – 1st Nov) = 1 (April) + 31 (May) + 30 (June) + 31 (July) + 31 (August)+30(September)+31(October)+1(November) = 186 days

Number of odd days from (30th April – 1st Nov)

$$= \text{Remainder of} \left(\frac{186}{7} \right) = 4 \text{ odd days}$$

Thus, day on 1st November will be = Monday + 4 = Friday.

And, 29th and 30th November will be Friday and Saturday, respectively.

So, in this case 5th Thursday is not possible in the month of November.

Case 2: When 30th April is 5th Monday

Total number of days from (1st May – 1st Nov) = 31 (May) + 30 (June) + 31 (July) + 31 (August) + 30 (September) + 31 (October) + 1 (November) = 185 days

Number of odd days from (1st May – 1st Nov)

$$= \text{Remainder of} \left(\frac{185}{7} \right) = 3 \text{ odd days}$$

Thus, day on 1st November will be = Monday + 3 = Thursday.

Then, 29th November will be the 5th Thursday of November.

Therefore, the birthday of the second child is on 29th November.

Thursdays in December will be on = 6th, 13th, 20th, 27th [Since, the days repeat after 7 days]

Hence, the birthday of the third child is on December 20th which is a Thursday.

Hence, option (b) is correct.

3. **The calendar for the year 2025 is same for**

(UPSC CSAT 2024)

(a) 2029   (b) 2030

(c) 2031   (d) 2033

Sol: Let 1 january 2025 - Monday

Then, 1 january 2026 - Monday + 1 = Tuesday [As, 2025 is an ordinary year]

Then, 1 january 2027 - Tuesday + 1 = Wednesday [As, 2026 is an ordinary year]

Then, 1 january 2028 - Wednesday + 1 = Thursday [As, 2027 is an ordinary year]

Then, 1 january 2029 - Thursday + 2 = Saturday [Since, 2028 is a leap year]

Then, 1 january 2030 - Saturday + 1 = Sunday [As, 2029 is an ordinary year]

Then, 1 january 2031 - Sunday + 1 = Monday [As, 2030 is an ordinary year]

Thus, calendar of 2025 will be same as that of 2031.

Hence, option (c) is correct.

TYPE 2: To find the day on a particular date when the reference date and day are not given

In order to find a particular day when reference is not given, the following steps are to be taken.

- **Step-1:** Choose the century leap year which is closest to the date you want to find. E.g. 1600, 2000 etc. The number of odd days for Century, which is divisible by 400, is 0.
- **Step-2:** Calculate the number of centuries you need to add to get to the preferred year. E.g 1900 = 1600 + 3 × 100. Add the odd days for the required centuries. For 300 centuries it is 1 odd day.
- **Step-3:** Calculate the number of leap years and ordinary years in the remaining date. e.g 5 feb 1969 = (preferred year = 1968), 1600 + 3 × 100 + 68. Therefore, 17 leap years and 51 ordinary years in 68 years. Add 2 odd days for every leap year and 1 odd day for every ordinary year.
- **Step-4:** Calculate the number of odd days till the preferred month and date. E.g. 5 February = 3 odd days of January and 5 odd days of February. In total, 8 odd days = 1 week + 1 odd day.
- **Step-5:** If odd day = 0/7, then the required day is Sunday

  If odd day = 1, then the required day is Monday

  If odd day = 2, then the required day is Tuesday

  If odd day = 3, then the required day is Wednesday

  If odd day = 4, then the required day is Thursday

  If odd day = 5, then the required day is Friday

  If odd day = 6, then the required day is Saturday

Now let us understand this through questions:

1. **What day is it on 24 Dec 2023?**

(a) Tuesday   (b) Thursday

(c) Sunday   (d) Saturday

Sol:

- We know that No. of odd days upto the century leap year = 0.

  So, Odd days up to the year 2000 are 0 odd days.

- Odd days for next 22 years = (5 leap years x 2 odd days) + (17 ordinary year x 1 odd day) = 10 + 17 = 27 odd days = 3 weeks + 6 odd days

- Odd days till 24 December = odd days in (January + February + March + April + May + June + July + August + September + October + November) + 24 days of December

   = 3 + 0 + 3 + 2 + 3 + 2 + 3 + 3 + 2 + 3 + 2 + 24

   = 26 + 24 odd days = 50 odd days = 7 weeks + 1 odd day

Total odd days = 0 + 6 + 1 = 7 odd days

$$= 1 \text{ week} + 0 \text{ odd day}$$

So, the day on 24th December 2023 will be a Sunday.

Hence, option (c) is correct.

2. **Which date of June 2099 among the following is Sunday?**   **(UPSC CSAT 2022)**

   (*a*) 4   (*b*) 5

   (*c*) 6   (*d*) 7

Sol: We know that 1st January, 2001 was a Monday. (*Just like 1st January, 1601, or 1st January, 1201, i.e. every 400 years).

In 100 years from 1st January, 2001 to 31st December, 2100, there will be 24 leap years (as 2100 is not a leap year) and 76 ordinary years.

We know that in an ordinary year the number of odd days is '1' while in each leap year the number of odd days is '2'.

So, the number of odd days from 1st January, 2001 to 31st December, 2100 = (24 × 2) + 76 = 48 + 76 = 124

In 124 days we will have 5 odd days.

[Since, 124 = 7 × 17 + 5]

So, the day on 1st January,

2101 must be Monday + 5 = Saturday.

So, the day on 1st January, 2100 must be Saturday – 1 = Friday [Since, there is 1 odd day in any non-leap (or ordinary) year]

So, the day on 1st January,

2099 must be Friday – 1 = Thursday

The number of odd days in the year 2099 are:

January – 3; February – 0; March – 3; April – 2; May – 3;

So, the number of odd days from 1st January, 2099 to 31st May, 2099 = 3 + 0 + 3 + 2 + 3 = 11 = 4

[Since, 11 = 7 × 1 + 4]

So, the day on 1st June, 2099 must be Thursday + 4 = Monday

So, the first Sunday in the month of June, 2099 will fall on the 7th.

Hence, option (d) is correct.

3. **Which day is 10ᵗʰ October, 2027?**

                                                            (UPSC CSAT 2021)

   (*a*) Sunday   (*b*) Monday

   (*c*) Tuesday   (*d*) Saturday

Sol: We need to find the number of odd days till 10th October 2027.

Number of odd days in 2000 years = 0. (odd days is 0 for every 400 years)

We know that in an ordinary year the number of odd days is '1' while in each leap year the number of odd days is '2'.

Number of odd days in 26 years = 6 leap years + 20 Ordinary years = 6 × 2 + 20 × 1 = 32 days = 4 × 7 + 4 = 4 weeks + 4 days = 4 odd days.

Number of odd days from 1/1/2027 to 10/10/2027 = 3 (Jan) + 0 (Feb) + 3 (March) + 2 (April) + 3 (May) + 2 (June) + 3 (July) + 3 (Aug) + 2 (Sep) + 10 (Oct) = 31 days

= 4 weeks + 3 days = 3 odd days

Total number of odd days till 10/10/2027 = 4 + 3 = 7 i.e. 0 odd days.

Here, we use odd days as:

'0 or 7' - Sunday

'1' - Monday

'2' - Tuesday

'3' - Wednesday

'4' - Thursday

'5' - Friday

'6' - Saturday

Therefore, the day on 10ᵗʰ October 2027 is Sunday.

Hence, option (a) is correct.

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Syllogism

Syllogism is a type of logical reasoning that derives conclusions based on two or more given premises. These premises are assumed to be true, and the conclusion logically follows from them.

Common Phrases used in Questions and Their Meaning

Common Phrase in Questions	Phrase Meaning	Example	Visual Representation
All A are B	Every member of A is also a member of B	All cats are animals.	animal / cats
No A is B	No member of A is a member of B	No dogs are cats.	cats dogs
Some A are B	At least one member of A is also in B	Some students are athletes.	students / Athletes
Some A are not B	At least one member of A is not in B	Some birds are not penguins.	Birds / penguins
Only A are B	All members of B are A, but not all A are B	Only birds are parrots (among the given).	Parrots / Birds
A is the same as B	A and B have the same members	The set of bachelors is the same as unmarried men.	Bachelors / unmarried

Common Premises and Their Logical Conclusions:

Type of Syllogism	Major Premise	Minor Premise	Logical Conclusion
Universal Affirmative	All A are B	All B are C	All A are C
Universal Negative	No A is B	All B are C	Some C are not A
Particular Affirmative	Some A are B	All B are C	Some A are C
Particular Negative	Some A are not B	All B are C	Cannot determine a conclusion
Mixed Affirmative	All A are B	Some B are C	Some A are C
Mixed Negative	No A is B	Some B are C	Some C may not be A
Existential	Some A are B	Some B are not C	Cannot determine a conclusion

Type 1: Two Or More Premise Arguments

In these types of questions 2 or more statements are given and we have to draw conclusions by those statements. Let us understand the method of solving these questions by the help of some questions:

1. **Consider the following Statements and Conclusions:**

   Statement-I: All cats are dogs.

   Statement-II: All cats are black.

   Conclusion-I: All dogs are black.

   Conclusion-II: Some dogs are not black.

   Which of the above Conclusions logically follows/ follow from the two given Statements, disregarding commonly known facts? **(UPSC CSAT 2020)**

   (*a*) Only Conclusion-I

   (*b*) Only Conclusion-II

   (*c*) Neither Conclusion-I nor Conclusion-II

   (*d*) Both Conclusion-I and Conclusion-II

Sol: **Step 1: Analyze the Statements**

**Statement-I:** All cats are dogs.

This means every cat is part of the group of dogs.

Statement-II: All cats are black.

This means every cat is black, but it says nothing about other animals, like dogs, being Black.

Step 2 : Make all the possible Venn diagrams

The given two statements can be presented in venn diagram as:

Case-1   Case-2
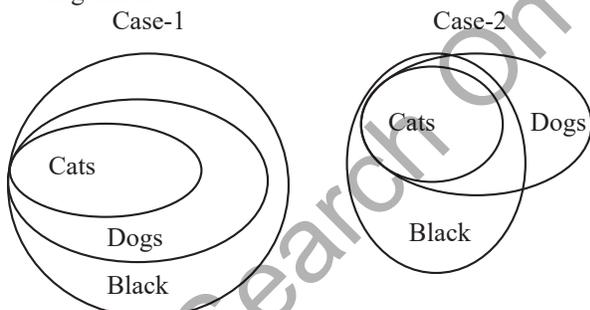
Step 2: Now Analyze the Conclusions

Conclusion-I: All dogs are black. This conclusion claims that every dog is black. However, from the statements, we only know that all cats (a subset of dogs) are black, not that all dogs are black. Hence, this conclusion does not follow.

Conclusion-II: Some dogs are not black. This conclusion claims that at least one dog is not black. However, the statements do not provide any information about dogs outside the group of cats. Since the blackness of other dogs is not addressed, this conclusion does not follow either.

Hence none of the conclusions follows.

Considering the above two cases, Neither the Conclusion-I nor the Conclusion-II follows.

Hence, option (c) is correct.

2. **Consider the following Statements and Conclusions:**

   Statement-I: Some rats are cats.

   Statement-II: Some cats are dogs.

   Statement-III: No dog is a cow.

   Conclusion-I: No cow is a cat.

   Conclusion-II: No dog is a rat.

   Conclusion-III: Some cats are rats.

   Which of the above conclusions is/are drawn from the statements? **(UPSC CSAT 2019)**

   (*a*) I, II and III   (*b*) Only I and II

   (*c*) Only III   (*d*) Only II and III

Sol: **Statement-I:** Some rats are cats.

This means there is at least some overlap between the groups of rats and cats.

Statement-II: Some cats are dogs.

This means there is at least some overlap between the groups of cats and dogs.

Statement-III: No dog is a cow.

This means the groups of dogs and cows have no overlap.

Now analyzing the conclusions:

Conclusion-I: No cow is a cat.

The statements do not provide any relationship between cows and cats. From the statements, we only know that no dog is a cow, but cows and cats are not mentioned together. Hence, this conclusion does not follow.

Conclusion-II: No dog is a rat.

The statements do not establish any relationship between dogs and rats. There is no information that connects these groups. Hence, this conclusion does not follow.

Conclusion-III: Some cats are rats.

This conclusion directly matches Statement-I, which says "Some rats are cats." Hence, this conclusion follows.

Possible Venn diagram:
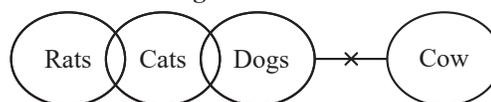
Conclusion-I is false as there is no direct link given between Cow and Cat.

Conclusion-II is false as there is no direct link between Dog and Rat.

Conclusion-III is true as some Cats are Rats.

Hence, option (c) is correct.

3. **Examine the following statements:**

I. None but students are the members of the club.

II. Some members of the club are married.

III. All married persons are invited for dance.

Which one of the conclusions can be drawn from the above statements? (UPSC CSAT 2012)

(*a*) All students are invited for dance.

(*b*) All married students are invited for dance.

(*c*) All members of the club are married persons.

(*d*) None of the above conclusions can be drawn.

Sol: **Conclusion (a) is incorrect:** We only know that married people are invited to the dance. Some students who are not married would not necessarily be invited.

Conclusion (b) is correct: Since some members (students) are married, and all married persons are invited to the dance, all married students will be invited to the dance.

Conclusion (c) is incorrect: We only know that some members are married, not all members. (given in statement 3)

Hence, option (b) is correct.

Type 2 : Evaluate The Accuracy of the Statements

Checking statement accuracy means finding two statements that cannot both be true at the same time but can both be false. To solve, carefully analyze each statement and look for contradictions or logical links.

Directions for the following 2 (two) items:

Each of the following two items consists of four statements. Of these four statements, two cannot both be true, but both can be false. Study the statements carefully and identify the two that satisfy the above condition. Select the correct answer using the codes given below each set of statements:

1. **Examine the following statements:**

I. All trains are run by diesel engine.

II. Some trains are run by diesel engine.

III. No train is run by diesel engine.

IV. Some trains are not run by diesel engine.

Codes: (UPSC CSAT 2011)

(*a*) I and II   (*b*) II and III

(*c*) I and III   (*d*) I and IV

Sol: Let's check the options one by one:

Option (a):

Statement I: All trains are run by diesel engine.

Statement II: Some trains are run by diesel engine.

If all trains are run by diesel engines (Statement I), then it is true that some trains are run by diesel engines (Statement II). So, statements I and II can be true together. Hence, option (a) cannot be the answer

Option (b):

Statement II: Some trains are run by diesel engine.

Statement III: No train is run by diesel engine.

It is not possible for some trains to be run by diesel engines (Statement II) while no train is run by diesel engines (Statement III). Hence, statements II and III cannot be true together. If statement II is false, it means no trains are run by diesel engines. Statement III also states that no train is run by a diesel engine. Therefore, statements II and III cannot both be false together, as they are inherently contradictory. Hence, option (b) cannot be the answer.

Option (c):

Statement I: All trains are run by diesel engine.

Statement III: No train is run by diesel engine.

If all trains are run by diesel engines (statement I), then it is not possible for no train to be run by diesel engines (statement III). Hence, statements I and III cannot be true together. Both statements I and III can be false together if the scenario is that only some trains are run by diesel engines (not all), and thus not no trains are run by diesel engines. Hence, option (c) can be the answer.

Hence, option (c) is correct.

2. **Examine the following statements:**

I. All animals are carnivorous.

II. Some animals are not carnivorous.

III. Animals are not carnivorous.

IV. Some animals are carnivorous.

Codes: (UPSC CSAT 2011)

(*a*) I and III   (*b*) I and II

(*c*) II and III   (*d*) III and IV

Sol: Let's check the options one by one:

Option (a): I and III

Statement I. All animals are carnivorous.

Statement III. Animals are not carnivorous.

These statements can't both be true because statement I says the opposite of statement III. However, both statements can be false if only some animals are carnivorous, making both statement I and statement III incorrect. Hence option (a) satisfies the condition given in question.

**Hence, option (a) is correct.**

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The Distance and Direction chapter focuses on understanding the relative positions and movements of objects in a two-dimensional plane. It enhances reasoning skills by solving problems based on directions (e.g., North, South, East, West) and distances between points.

KEY CONCEPTS

- 4 primary directions: North, South, East, and West
- 4 intermediate directions: North - East(NE), North - West (NW), South - West (SW) and South - East (SE).
- The Sun rises in the East, and sets in the West. Before noon the shadow will fall towards west and after noon it will fall in the east direction.
- If you are facing North, then West will be on your left side, the East on your right, and the South will be directly behind you.

Following diagram will help us in better understanding of the direction:
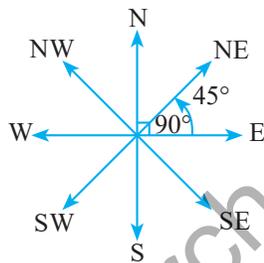
Shortest Distance:

To find the shortest distance between two points we use Pythagoras theorem:
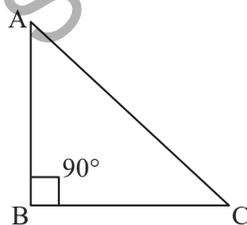
Here, AB = perpendicular, BC = Base, AC = Hypotenuse, Angle B = 90°

Now, by Pythagoras theorem shortest distance between points A and C:

$$AC = \sqrt{(AB)^2 + (BC)^2}$$

Important Points to remember while solving the problems:

- Diagrams are crucial in solving direction and distance problems. They help in visualizing the movement and turns.
- While plotting movements on a diagram, it is essential to mark the starting point, the path taken, turns (right or left), and the final point.
- Be careful while taking a left turn (Anti-clockwise turn) and Right turn (Clockwise turn).
- Calculate distances using Pythagoras's theorem in problems involving diagonal movements (e.g., NE, NW, SE, SW).
- Mark the distance along with the line of direction.

TYPES OF QUESTIONS

There are mainly four types of questions which are usually asked in the exam:

TYPE 1: Final direction questions

In this type of problem, the goal is to trace the path taken from the start and determine the final direction or orientation relative to the starting point.

1. **Rohan is facing the West. He turns 45° in the clockwise direction and then another 180° in the same direction and then 270° in the anti-clockwise direction. Which direction is he facing now?**

   (*a*) South   (*b*) West

   (*c*) North-West   (*d*) South-West

Sol:

The movement of Rohan is shown in the figure:
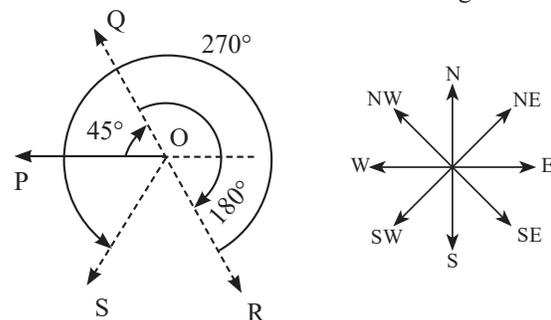
Rohan is standing at point O facing towards the west along line OP. According to the question after the 1st turn of 45° in clockwise direction his direction is along the line OQ. After his 2nd turn of 180° in the same direction his direction is along the line OR.

After his final turn of 270° in the anticlockwise direction, the direction of Rohan is along the line OS which is the south west direction.

Hence, option (d) is correct.

2. A person walks 100 m Westward, then turns left and walks 100 m. He then takes 225° turn clockwise. In which direction is he walking now?

(UPSC CSAT 2024)

(a) South-West   (b) South-East
(c) North-West   (d) North-East

Sol: The following diagram can be made based upon the information given in the question.
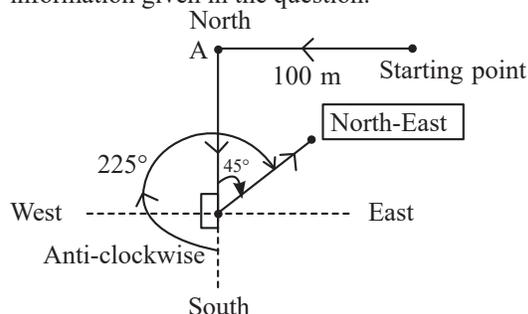
Thus, it is clear from the diagram that he is walking in the North-East direction.

Hence, option (d) is correct.

TYPE 2: Displacement Questions

In this type of problem, the focus is on calculating the shortest distance between the starting position and final position:

1. Two friends 'X' and 'Y' start running and they run together for 50 m in the same direction and reach a point. 'X' turns right and runs 60 m, while Y turns left and runs 40 m. Then 'X' turns left and runs 50 m and stops, while 'Y' turns right and runs 50 m and then stops. How far are the two friends from each other now?

(UPSC CSAT 2022)

(a) 100 m   (b) 90 m
(c) 60 m   (d) 50 m

Sol: Route of X and Y is shown in the figure below:
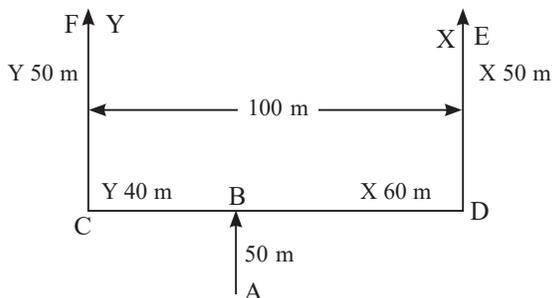
Now according to question X and Y start running from point A and run till point B

AB = 50m, After reaching B, X turn right and reached at point D such that

BD = 60m

After reaching point B, Y turn left and reached point C such that

BC = 40m, After reaching point C, Y turn left and reach point F such that

CF = 50 m

After reaching point D, X take Right and reach point E such that DE = 50 m, Finally X and Y are T E and F respectively to find distance between X and Y we need to find EF (distance between E and F)

EF = BC + BD EF

= 60 + 40 = 100m

Hence, option (a) is correct.

2. 'A' started from his house and walked 20 m towards East, where his friend B joined him. They together walked 10 m in the same direction. Then 'A' turned left while 'B' turned right and travelled 2 m and 8 m, respectively. Again 'B' turned left to travel 4 m followed by 5 m to his right to reach his office. 'A' turned right and travelled 12 m to reach his office.

What is the shortest distance between the two offices?

(UPSC CSAT 2019)

(a) 15 m   (b) 17 m
(c) 19 m   (d) 20 m

Sol: The given information can be depicted as:
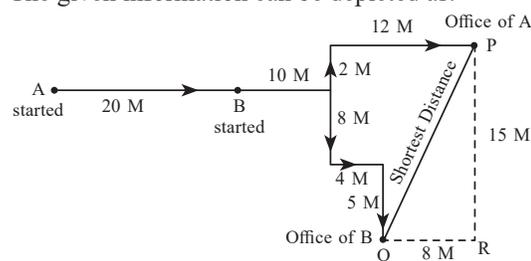
We need to find the distance between 'P' and 'Q'

Using the Pythagorean theorem, we have

In right triangle PRQ

$(QP)^2 = (QP)^2 + (QP)^2 = 8^2 + 15^2 = 64 + 225 = 289$

So, $QP = 17$

Thus, the shortest distance between offices of 'A' and 'B' is 17 km.

Hence, option (b) is correct.

## TYPE 3: Direction and Displacement combined questions

These types of questions involve a mix of direction tracking and distance calculation:

1. **A person *X* was driving in a place where all roads ran either North-South or East-West, forming a grid. Roads are at a distance of 1 km from each other in parallel. He started at the intersection of two roads, drove 3 km North, 3 km West and 4 km South. Which further route could bring him back to his starting point, if the same route is not repeated?**

   (UPSC CSAT 2016)

   (*a*) 3 km East, then 2 km South
   (*b*) 3 km East, then 1 km North
   (*c*) 1 km North, then 2 km West
   (*d*) 3 km South, then 1 km North

Sol:
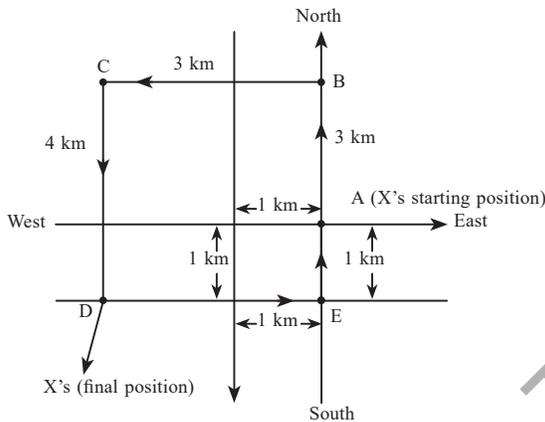
If '*X*' takes 3 km East from '*D*' to *E* and then 1 km North, then he will reach point '*A*' again.

Hence, option (b) is correct.

2. **Rajesh left his home for the office in the car. He drove 15 km straight towards North and then turned Eastwards and covered 8 km. He then turned left and covered 1 km. He again turned left and drove for 20 km and reached office. How far and in what direction is his office from the home?**

   (*a*) 21 km West   (*b*) 15 km North-East
   (*c*) 20 km North-West   (*d*) 26 km North-West

Sol: Movement of Rajesh is shown in the figure below:
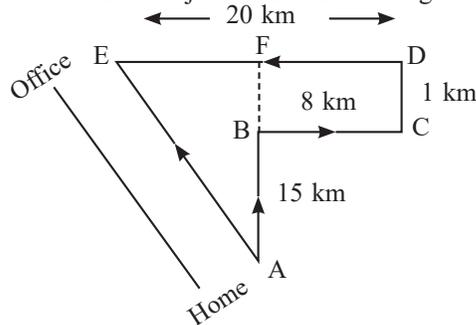
Home of Rajesh is at point A and his office is at point E.

We need to find AE in the triangle AEF.

In triangle AEF, AF = 15 + 1 =16 km

And, EF = 12km.

$AE = \sqrt{(16)^2 + (12)^2} = \sqrt{400} = 20$ km

And it is clear from the figure that the direction of the office is north west from the home.

Hence, option (c) is correct.

TYPE 4: Shadow based questions

This type of questions focus on determining directions based on the position of shadows at different times of the day.

Shadow formation during sunrise
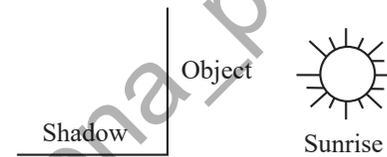
Shadow formation during sunset
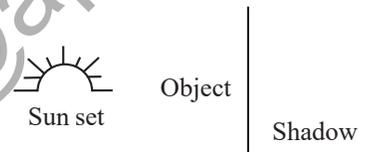
1. **One morning after Sunrise, Ram and Shyam were standing at Shivaji crossing in Delhi with their backs opposite to each other. Ram's shadow fell exactly towards his right hand side. Which direction was Shyam facing?**

   (*a*) East   (*b*) West
   (*c*) North   (*d*) South

Sol: The Direction Diagram of Ram and Shyam are as follows:
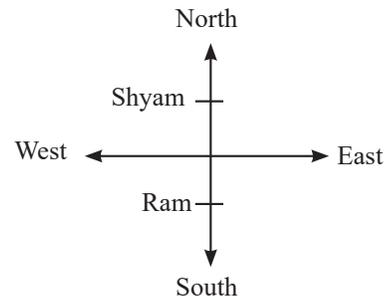
As Ram's shadow fell exactly towards his right hand side, therefore he faced South. According to the question, Shyam's direction was opposite to that of Ram's, therefore Shyam was facing North.

Hence, option (c) is correct.

❖   ❖   ❖

KEY CONCEPTS

- If present age of a person is 'x' years, then

  Age after 'n' years = $(x + n)$ years

  Age 'n' years ago = $(x - n)$ years

- If present average age of a group of 'n' persons is 'm' years and 'p' persons having average age 'x' years joins the group, then new average age of the group = $\dfrac{n \times m + p \times x}{n + p}$ years

- If present ages of 'A' and 'B' is 'M' years and 'N' years, respectively and the ratio of their ages after 'n' years becomes $p : q$, then $\dfrac{M + n}{N + n} = \dfrac{p}{q}$

- If ratio of present ages of 'A' and 'B' is $m : n$, respectively and after 't' years the ratio of their ages will become $a : b$, then

- First, we are required to suppose that the present ages of 'A' and 'B' is 'mx' years and 'nx' years, respectively

  So, $\dfrac{mx + t}{nx + t} = \dfrac{a}{b}$

Let us understand the method to solve questions based on these concepts:

1. **A father said to his son, "n years back I was as old as you are now. My present age is four times your age n years back". If the sum of the present ages of the father and the son is 130 years, what is the difference of their ages?** **(UPSC CSAT 2024)**

   (*a*) 30 years (*b*) 32 years

   (*c*) 34 years (*d*) 36 years

Sol: Let present age of father and son be 'F' years and 'S' years, respectively.

According to question;

F – n = S

Or, F = n + S …(*i*)

And, F = 4(S – n)

Or, $\left(\dfrac{F}{4}\right)$ = S – n ...(*ii*)

Adding equation (i) and (ii), we ge;

$F + \left(\dfrac{F}{4}\right) = 2S$

Or, $\left(\dfrac{5F}{4}\right) = 2S$

Or, $\left(\dfrac{F}{S}\right) = \left(\dfrac{8}{5}\right)$

So, Age of Father = $\dfrac{8}{8+5} \times 130 = \left(\dfrac{8}{13}\right) \times 130$ = 80 years

And, age of Son = 130 – 80 = 50 years

Required difference = 80 – 50 = 30 years

Hence, option (a) is correct.

2. **The average age of a teacher and three students is 20 years. If all the three students are of the same age and the difference between the age of the teacher and each student is 20 years, then what is the age of the teacher?** **(UPSC CSAT 2020)**

   (*a*) 25 years (*b*) 30 years

   (*c*) 35 years (*d*) 45 years

Sol: Let the age of each student be 'x' years and the age of the teacher be 'y' years.

According to the question

$3x + y = 20 \times 4$

$\Rightarrow 3x + y = 80$ ...(*i*)

Also, $y - x = 20$ ...(*ii*)

By subtracting equation (*ii*) from equation (*i*), we get;

$(3x + y) - (y - x) = 80 - 20$

Or, $4x = 60$

$\Rightarrow x = 15$

So, $y = x + 20 = 15 + 20 = 35$

Therefore, the age of the teacher is 35 years.

Hence, option (c) is correct.

8 Sitting Arrangement

LINEAR SITTING ARRANGEMENT

In the Linear Arrangement type of puzzles, you need to arrange objects or people in a straight line. This line can be either horizontal (side by side) or vertical (one above the other). The challenge here is to figure out where each object or person goes in the line, based on clues you're given. These clues can tell you:

- **Exact Position:** Where exactly an object or person needs to be placed in the line.
- **Relative Position:** Where an object or person should be in relation to another.

The following facts are necessary to solve these kinds of questions:

1. If A, B, C and D are facing toward the north direction and P, Q, R and S are facing toward the south direction in a line then positions at their left and the right will be as shown in the following figures:

Northward facing

Southward facing
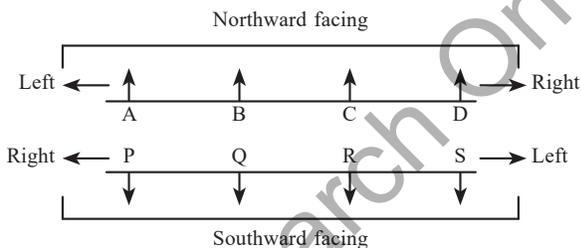
CIRCULAR SITTING ARRANGEMENT

Facing the centre

Facing the direction opposite to centre
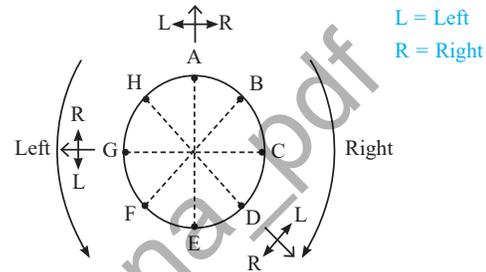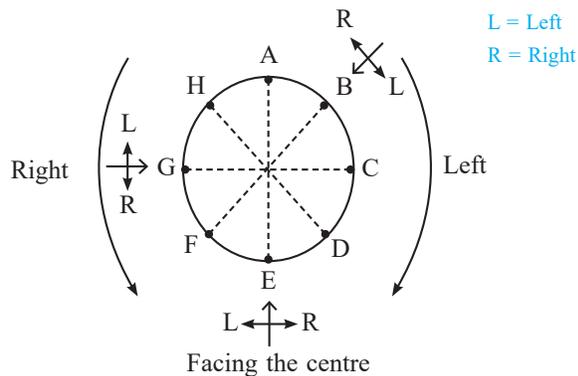
Let us understand the method to solve questions based on these concepts:

1. **40 children are standing in a circle and one of them (say child-1) has a ring. The ring is passed clockwise. Child-1 passes on to the child-2, child-2 passes on to child-4, child-4 passes on to child-7 and so on. After how many such changes (including child-1) will the ring be in the hands of child-1 again?**

   (UPSC CSAT 2023)

   (*a*) 14   (*b*) 15
   (*c*) 16   (*d*) 17

Sol: According to the question, the ring is passed clockwise starting from child 1, then child 2, then child 4, then child 7, and so on.

Let us represent child 1 as C1, child 2 as C2 and so on

So, the series is:

C1, C2, C4, C7, C11, C16, C22, C29, C37, C6, C16, C27, C39, C12, C26, C1
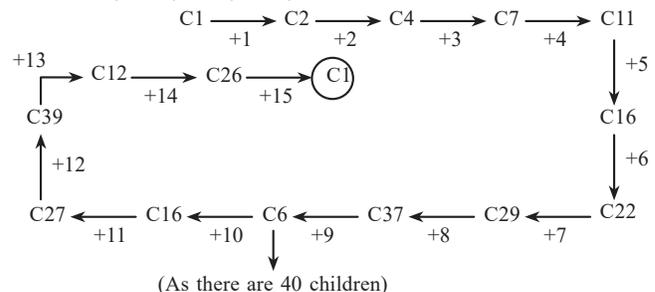
(As there are 40 children)

So, after 15 changes the ring will again be in the hands of child 1.

Hence, option (b) is correct.

2. **Five people 'A', 'B', 'C', 'D' and 'E' are seated about a round table. Every chair is spaced equidistant from adjacent chairs.**

(i) **'C' is seated next to 'A'.**

(ii) **'A' is seated two seats from 'D'.**

(iii) **'B' is not seated next to 'A'.**

On the basis of above information, which of the following must be true? **(UPSC CSAT 2016)**

I. 'D' is seated next to 'B'.

II. 'E' is seated next to 'A'.

III. 'D' and 'C' are separated by two seats.

Select the correct answer using the codes given below.

(a) Only I   (b) I and II

(c) Only III   (d) Neither I nor II nor III

Sol: The arrangement is in a circular table.

'C' is seated next to 'A'

Conclusion: _AC_ or _CA_

'A' is seated two seats from 'D'

Conclusion: A_D or D_A

'B' is not seated next to 'A'

Conclusion: 'E' must be next to 'A'.

Combining, ACDBEA

So, the following arrangements can be possible

A
C   E or E   C
D   B   B   D
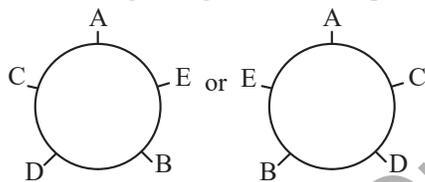
Therefore, I and II are correct.

Hence, option (b) is correct.

3. **Four children are sitting in a row. 'A' is occupying the seat next to 'B' but not next to 'C'. If 'C' is not sitting next to 'D', who is/are occupying seat/seats adjacent to 'D'?**   **(UPSC CSAT 2014)**

(a) B   (b) A

(c) B and A   (d) Impossible to tell

Sol: The arrangement, as per the given information, is possible only if 'C' is sitting next to 'B' and 'D' is sitting next to 'A'.

So, the two possible arrangements are

Possibility '1':

C	B	A	D

Possibility '2':

D	A	B	C

Clearly, only 'A' is sitting adjacent to 'D'.

Hence, option (b) is correct.

9. **Five people A, B, C, D and E are, seated about a round table, Every chair is spaced equidistant from adjacent chairs,**

I. C is seated next to A.

II. A is seated two seats from D.

III. B is not seated next to A.

Which of the following must be true?

(UPSC CSAT 2013)

I. D is seated next to B.

II. E is seated next to A.

Select the correct answer from the codes given below:

(a) I only   (b) II only

(c) Both I and II   (d) Neither I nor II

Sol: We can understand the information given in question by using following diagrams:

A   C
    C   or   A
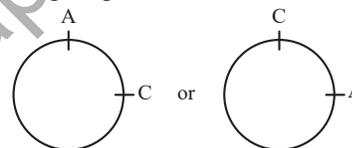
C is seated next to A: there are two possibilities as shown in the diagram.

We can take any of these possibilities.

Now, A is seated two seats from D and B is not seated next to A.

It implies there is atleast one person between A and B, and there is atleast one person between A and D.

So it implies that E will be seated next to A. The arrangements can be:

E   E
B   A   Or   D   A
D   C   B   C
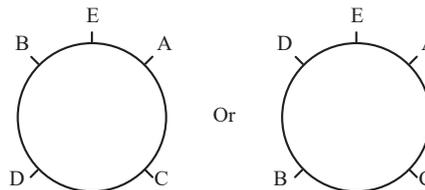
Now, from figure it is clear that D is seated next to B and E is seated next to A.

So, both the statements are correct.

Hence, option (c) is correct.

❖   ❖   ❖

KEY FORMULA

- Total Number of persons in Queue/Row = Position from Left + Position from Right – 1
- Rank/Position from Left/Top = Total – Rank from Right/Bottom + 1
- Rank/Position from Right/Bottom = Total – Rank from Left/Top + 1

Let us understand the method to solve questions based on these concepts:

1. In a class of 60, where girls are twice that of boys, Kamal ranked seventeenth from the top. If there are 9 girls ahead of Kamal, how many boys are after him in rank? **(UPSC CSAT 2016)**

    (*a*) 13   (*b*) 12

    (*c*) 7   (*d*) 3

Sol: Let the number of boys and girls in the class be '*x*' and '2*x*', respectively

According to question;

$x + 2x = 60 \Rightarrow x = 20$

So, number of boys = x =20

And, number of girls = 2x = 40

If Kamal is at 17th position and 9 girls are ahead of him, then out of 16 students are ranked ahead of him, 7 (= 16 – 9) are boys.

Therefore, there are 12 [= (20 – 7 – 1)] boys ranked after him.

Hence, option (b) is correct.

2. In a row 'A' is in the 11th position from the left and 'B' is in the 10th position from the right. If 'A' and 'B' interchange, then 'A' becomes 18th from the left. How many persons are there in the row other than 'A' and 'B'? **(UPSC CSAT 2014)**

    (*a*) 27   (*b*) 26

    (*c*) 25   (*d*) 24

Sol: **Step 1:** Total positions in the row

'A' is in the 11th position from the left.

'B' is in the 10th position from the right.

When 'A' and 'B' interchange positions:

'A' becomes 18th from the left.

Step 2: Total number of persons in the row

If 'A' is now 18th from the left, and 'B' was originally 10th from the right, we can calculate the total number of positions in the row using the following formula:

Total number of persons = Position of A from the left after interchange + Initial position of B from the right − 1

Total number of persons = 18 + 10 – 1 =27

Step 3: Persons other than 'A' and 'B'

To find the number of persons other than 'A' and 'B', subtract 2 (for 'A' and 'B') from the total:

Persons other than 'A' and 'B' = 27 – 2 = 25

Alternative solution:

Since, '*A*' is 11th from left. So, there must be 10 persons in the left of '*A*'

Also, '*B*' is 10th from the right. So, there must be 9 persons in the right of '*B*'.

Let number of persons between '*A*' and '*B*' be '*x*'

The pictorial representation of the same is:
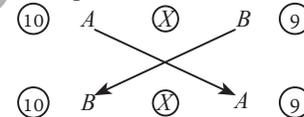
Now, after exchange '*A*' becomes 18th from the left

Thus, 10 + B + x + A = 18

Or, $x = 18 – 12 = 6$

So, there are 6 persons between '*A*' and '*B*'.

And, total number of persons except '*A*' and '*B*'

= 10 + 6 + 9 = 25

Hence, option (c) is correct.

3. In the game of wrestling Vikram performs better than Rohan but is not as good as Arjun, who is outperformed by Suresh. Gaurav is not as good as Suresh but is better than Arjun. How many players are better than Gaurav?

    (*a*) one   (*b*) two

    (*c*) three   (*d*) Data Inadequate

Sol: Vikram is better than Rohan but not as good as Arjun:

This tells us the relative ranking of Vikram, Rohan, and Arjun: Arjun > Vikram > Rohan.

Arjun is outperformed by Suresh:

Adding this to our previous ranking, we get:

Suresh > Arjun > Vikram > Rohan.

Gaurav is not as good as Suresh but is better than Arjun:

This means Gaurav falls between Suresh and Arjun in the ranking:

Suresh > Gaurav > Arjun > Vikram > Rohan.

Based on our ranking: Suresh is the only player better than Gaurav. So answer will be one.

Hence, option (a) is correct.

A Venn Diagram is a graphical illustration of mathematical or logical relationships. It uses shapes, usually circles, to represent sets. The position and overlap of these circles demonstrate how the sets intersect and relate to one another.

MAIN PARTS OF VENN DIAGRAMS
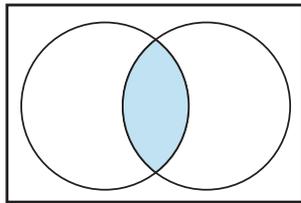
- **Sets:** Circles in the diagram. Each set is a group of things that have something in common.
- **Universal Set:** The big box that holds all the sets. It has all the items we are interested in.
- **Intersection:** Where circles overlap. It shows items that are in both groups.
- **Union:** All the space in the circles together. It covers all items in any of the sets.
- **Complement:** The space outside a circle but inside the rectangle. It shows items not in that set.

Let us understand this with the help of following examples:

1. **In the above figure, circle P represents hard working people, circle Q represents intelligent people, Circle R represents truthful people and circle represents honest people. Which region represents the people who are intelligent, honest and truthful but not hardworking?** **(UPSC CSAT 2012)**
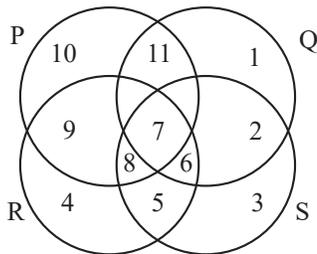
(*a*) 6   (*b*) 7
(*c*) 8   (*d*) 11

Sol: P – Hardworking people, Q – Intelligent people R – Truthful people, S – Honest people

We have to find the region that represents the people who are intelligent, honest and truthful but not hardworking i.e. the region includes Q, R and S but excludes P.

That is 6 people.

Hence, option (a) is correct.

2. **Consider the following Venn diagram:**
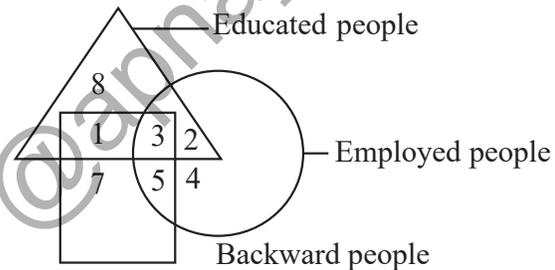
How many educated people are backward people?

(*a*) 4   (*b*) 6
(*c*) 5   (*d*) 7

Sol: Triangle shows the number of Educated people and rectangle represents the backward people where circle represents employed people.

Number of people which are educated and backward is the intersection of triangle and rectangle Hence, the number of people who are educated and backward = 1 + 3 = 4.

Hence, option (a) is correct.

SET THEORY USING VENN DIAGRAM

Set theory is a fundamental branch of mathematics that studies collections of objects called sets. Venn diagrams provide an intuitive way to visualize sets and their relationships using overlapping shapes, often circles. These diagrams simplify understanding concepts like union, intersection, and complement by showing overlaps and differences visually

Venn Diagram for Two Variables

There are 'N' number of people in a village such that each of them like at least one of the two phones i.e. android or apple. The representation of this type of data can be given as follows:
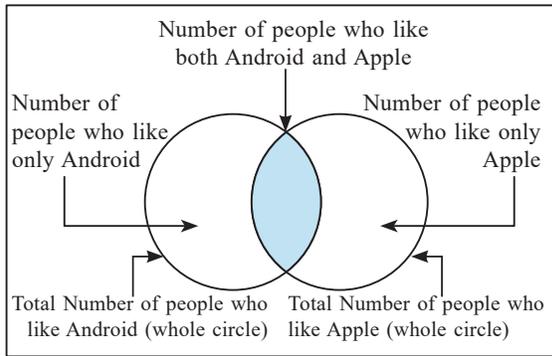
Total number of people in the village =

N = Number of people who like only Android + Number of people who like only Apple + Number of people who like both Android as well as Apple

$n(A)$ = Total number of people who like Apple

$n(A)$ = (Number of people who like only Apple)
$$+ \, n(A \cap B)$$

$n(B)$ = Total number of people who like Android

$n(B)$ = (Number of people who like only Android)
$$+ \, n(A \cap B)$$

$n(A \cap B)$ = Number of people who like both Apple and Android

$n(A \cup B)$ = Total number of people in the village = N

So, $n(A \cup B) = n(A) + n(B) - n(A \cap B)$

Let us understand the method to solve questions based on these concepts:

1. **There are 50 students admitted to a nursery class. Some students can speak only English and some can speak only Hindi. 10 students can speak both English and Hindi. If the number of students who can speak English is 21, then how many students can speak Hindi, how many can speak only Hindi and how many can speak only English?** **(UPSC CSAT 2014)**

   (*a*) 21, 11 and 29 respectively

   (*b*) 28, 18 and 22 respectively

   (*c*) 37, 27 and 13 respectively

   (*d*) 39, 29 and 11 respectively

Sol: Number of English speakers only = Total number of English speakers – Number of speakers of both Hindi and English

   = 21 – 10 = 11

   Number of Hindi speakers = Total students – Number of English speakers only

   = 50 – 11 = 39

Number of Hindi speakers only = Total number of hindi speakers – Number of speakers of both Hindi and English

= 39 – 10 = 29

Required answer is (39, 29, 11)

Total number of students = 50
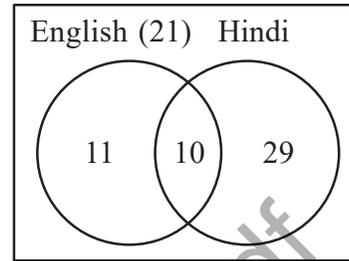
Hence, option (d) is correct.

2. **All members of a club went to Mumbai and stayed in a hotel. On the first day, 80% went for shopping and 50% went for sightseeing, whereas 10% took rest in the hotel.**

   Which of the following conclusion(s) can be drawn from the above data?

   I. 40% members went for shopping as well as sightseeing.

   II. 20% members went for only shopping.

   Select the correct answer using the code given below:

   (*a*) I only   (*b*) II only

   (*c*) Both I and II   (*d*) Neither I nor II

Sol: We know: $n(A \cup B) = n(A) + n(B) - n(A \cap B)$

   Let the total number of people be 100.

   People who went for shopping = 80% of 100 = 80 = $n(A)$

   People went for who went for sightseeing = 50% of 100 = 50 = $n(B)$

   People who took rest = 10% of 100 = 10

   People who went for shopping or sightseeing or both = 100 – 10 = 90 = $n(A \cup B)$

   Therefore,

   $90 = 80 + 50 - n(A \cap B)$

   $n(A \cap B) = 130 - 90 = 40$

   People who went for shopping as well as sightseeing = 40

   People who went for only shopping = 80 – 40 = 40

   People who went for only sightseeing = 50 – 40 = 10

Total number of people who went for shopping and sightseeing = 90

Shopping   Sightseeing

40   10   40
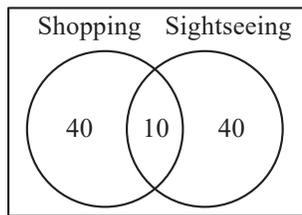
Since, percentage of members who went for shopping as well as sightseeing = $\dfrac{40}{100} \times 100 = 40\%$

So, conclusion-I is true.

Percentage of number of members who went only for shopping = $\dfrac{40}{100} \times 100 = 40\%$

So, conclusion-II is false.

Hence, option (a) is correct.

3. **Out of a total of 120 musicians in a club, 5% can play all three instruments, guitar, violin and flute. It so happens that the number of musicians who can play any and only two of the above instruments is 30. The number of musicians who can play guitar alone is 40. What is the total number of those who can play violin alone and flute alone?** (UPSC CSAT 2014)

(a) 45   (b) 44

(c) 38   (d) 30

Sol:

Guitar   Violin

A   B   F

G

C   E

D
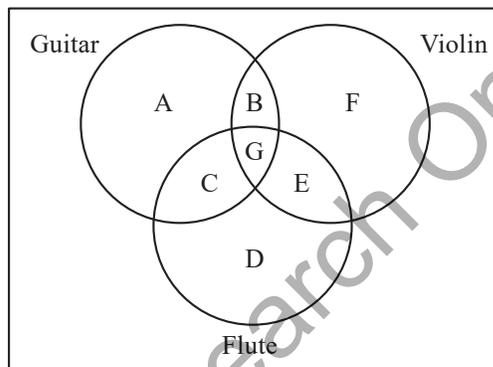
Flute

Given, 'a' = 40

'g' = 5% of 120 = 6

The number of musicians who can play any two and only two of the above instruments is 30.

i.e. $b + c + e = 30$

Number of musicians who can play only violin or only flute $= d + f$

Since, $a + b + c + d + e + f + g = 120$

$40 + 30 + 6 + d + f = 120$

$d + f = 120 - 76 = 44$

Thus, the number of musicians who can play violin or flute alone = 44

Hence, option (b) is correct.

4. **In a town, 45% population read magazine 'A', 55% read magazine 'B', 40% read magazine 'C', 30% read magazines 'A' and 'B', 15% read magazines 'B' and 'C', 25% read magazines 'A' and 'C'; and 10% read all the three magazines. What percentage do not read any magazine?** (UPSC CSAT 2015)

(a) 10%   (b) 15%

(c) 20%   (d) 25%

Sol: Let $n(A)$ = population who read magazine 'A'

Let $n(B)$ = population who read magazine 'B'

Let $n(C)$ = population who read magazine 'C'

$n(A \cap B)$ = population who read both magazine 'A' and 'B'

$n(A \cap C)$ = population who read both magazine 'A' and 'C'

$n(B \cap C)$ = population who read both magazine 'B' and 'C'

$n(A \cap B \cap C)$ = population who read all three magazines.

$n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B) - n(A \cap C)$
$\qquad\qquad - n(B \cap C) + n(A \cap B \cap C)$

$= 45\% + 55\% + 40\% - 30\% - 25\% - 15\% + 10\% = 80\%$

MAGAZINE 'A' [45%]   MAGAZINE 'B' [55%]

10%   20%   20%

10%

15%   5%

10%
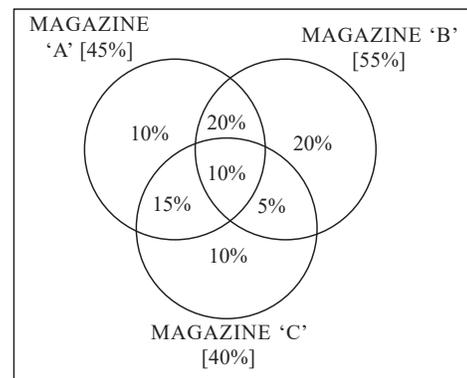
MAGAZINE 'C' [40%]

Population who don't read any magazine

$= 100\% - n(A \cup B \cup C) = 100\% - 80\% = 20\%$

Hence, option (c) is correct.

❖   ❖   ❖

INEQUALITIES AND CONCLUSIONS

Inequality	Conclusion
$A > B > C$	$A > C$
$A \geq B > C$	$A > C$
$A > B \geq C$	$A > C$
$A = B > C$	$A > C$
$A > B = C$	$A > C$
$A \geq B = C$	$A \geq C$ or $A > C$ or $A = C$
$A = B \geq C$	$A \geq C$ or $A > C$ or $A = C$
$A \geq B \geq C$	$A \geq C$ or $A > C$ or $A = C$

Let us understand the method to solve questions based on these concepts:

1. **Consider the following:**

   I. A + B means 'A' is neither smaller nor equal to 'B'.

   II. A – B means 'A' is not greater than 'B'.

   III. A × B means 'A' is not smaller than 'B'.

   IV. A ÷ B means 'A' is neither greater nor equal to 'B'.

   V. A ± B means 'A' is neither smaller nor greater than 'B'.

   Statement: $P \times Q$, $P - T$, $T \div R$, $R \pm S$

   Conclusion-I: $Q \pm T$

   Conclusion-II: $S + Q$

   Which one of the following is correct in respect of the above Statement and the Conclusions?

   (UPSC CSAT 2023)

   (*a*) Only Conclusion-I follows from the Statement.

   (*b*) Only Conclusion-II follows from the Statement.

   (*c*) Both Conclusion-I and Conclusion-II follow from the Statement.

   (*d*) Neither Conclusion-I nor Conclusion-II follows from the Statement

   Sol: It is given that,

   $A + B$ means $A > B$

   $A - B$ means $A \leq B$

   $A \times B$ means $A \geq B$

   $A \div B$ means $A < B$

   $A \pm B$ means $A = B$

   So, $P \times Q$ means $P \geq Q$

   $P - T$ means $P \leq T$

   $T \div R$ means $T < R$

   $R \pm S$ means $R = S$

Combining all the relations, we have;

$S = R > T \geq P \geq Q$

Conclusion-I: $Q \pm T$ or $Q = T$. This is not necessarily true as $T \geq P \geq Q$ or $T \geq Q$

Conclusion-II: $S + Q$ or $S > Q$. This is true as $S = R > T \geq P \geq Q$ or $S > Q$

Hence, option (b) is correct.

2. **If 'x' is greater than or equal to 25 and 'y' is less than or equal to 40, then which one of the following is always correct?** **(UPSC CSAT 2019)**

   (*a*) 'x' is greater than 'y'

   (*b*) $(y - x)$ is greater than 15

   (*c*) $(y - x)$ is less than or equal to 15

   (*d*) $(x + y)$ is greater than or equal to 65

   Sol: **Given,**

   $x \geq 25$ and $y \leq 40$

   Option (*a*): 'x' is greater than 'y'

   For $x = 26$ and $y = 39$

   $\Rightarrow x < y$ [As, 26 < 39]

   So, option (*a*) is incorrect.

   Option (*b*): $(y - x)$ is greater than 15

   For $y = 40$ and $x = 40 \Rightarrow (y - x) = 0$

   So, option (*b*) is incorrect.

   Option (*c*): $(y - x)$ is less than or equal to 15

   $x \geq 25$ and $y \leq 40$ (given)

   For maximum value of $(y - x)$, take the maximum value of 'y' and minimum value of 'x'

   i.e., $y = 40$ and $x = 25$

   Thus, maximum value of $(y - x) = 15$

   Thus, we can say that $(y - x)$ is less than or equal to 15

   Hence, option (c) is correct.

3. **Consider the following statements**

   I. Either 'A' and 'B' are of the same age or 'A' is older than 'B'.

   II. Either 'C' and 'D' are of the same age or 'D' is older than 'C'.

   III. 'B' is older than 'C'.

   Which of the following conclusions can be drawn from the above statements? **(UPSC CSAT 2016)**

   (*a*) 'A' is older than 'B'   (*b*) 'B' and 'D' are of the same age

   (*c*) 'D' is older than 'C'   (*d*) 'A' is older than 'C'

Sol: According to the given statements, we have

   I. Either $A = B$ or $A > B$

  II. Either $C = D$ or $D > C$

 III. $B > C$

So, we have $A \geq B > C \leq D$

For option (a): 'A' is older than 'B' is not definitely true as $A = B$

For option (b): 'B' and 'D' are of the same age is not definitely true as $B > C \leq D$

For option (c): 'D' is older than 'C' is not definitely true as $D = C$

For option (d): 'A' is older than 'C' is definitely true as $A \geq B > C$

Hence, option (d) is correct.

4. **If A runs less fast than B, and B runs as fast but not faster than C; then, as compared to A, C runs:**

                                    (UPSC CSAT 2015)

  (a) slower than A

  (b) faster than A

  (c) with the same speed as A

  (d) Given data is not sufficient to determine

Sol: Let 'A' runs less fast than 'B' is written as $A < B$

And, $B \leq C$

So, the final possible relation is:

$A < B \leq C$

Thus, we can say that 'C' runs faster than 'A'

Hence, option (b) is correct.

5. **Consider the following statements:**

    I. 'A' is older than 'B'.

   II. 'C' and 'D' are of the same age.

  III. 'E' is the youngest.

  IV. 'F' is younger than 'D'.

   V. 'F' is older than 'A'.

How many statements given above are required to determine the oldest person/persons?

                                      (UPSC CSAT 2023)

  (a) Only two   (b) Only three

  (c) Only four   (d) All five

Sol: Let us make a representation of the given statements

Statement-I: $A > B$

Statement-II: $C = D$

Statement-III: $E < A, B, C, D$ and F

Statement-IV: $F < D$

Statement-V: $F > A$

Combining, all the statements, we have;

$C = D > F > A > B > E$

Thus, we need all the five statements to answer the question.

Hence, option (d) is correct.

6. **If 'x' is greater than or equal to 25 and 'y' is less than or equal to 40, then which one of the following is always correct?** **(UPSC CSAT 2019)**

  (a) 'x' is greater than 'y'

  (b) $(y - x)$ is greater than 15

  (c) $(y - x)$ is less than or equal to 15

  (d) $(x + y)$ is greater than or equal to 65

Sol: Given,

$x \geq 25$ and $y \leq 40$

Option (a): 'x' is greater than 'y'

For $x = 26$ and $y = 39$

$\Rightarrow x < y$ [As, $26 < 39$]

So, option (a) is incorrect.

Option (b): $(y - x)$ is greater than 15

For $y = 40$ and $x = 40$

$\Rightarrow (y - x) = 0$

So, option (b) is incorrect.

Option (c): $(y - x)$ is less than or equal to 15

$x \geq 25$ and $y \leq 40$ (given)

For maximum value of $(y - x)$, take the maximum value of 'y' and minimum value of 'x'

i.e., $y = 40$ and $x = 25$

Thus, maximum value of $(y - x) = 15$

Thus, we can say that $(y - x)$ is less than or equal to 15

Hence, option (c) is correct.

❖   ❖   ❖

CUBE

If a larger cube of edge length 'M' units is painted from all sides and is cut into 'n' identical smaller cubes of edge length 'N' units, then the number of cubes along each edge of the large cube is $= n = \dfrac{M}{N}$

- Total number of cubes $= n^3$
- Number of cubes with zero or no face painted = Cubes which are in second layer or inside $= (n-2)^3$
- Number of cubes with exactly one face painted = Cubes on the surface or face $= 6(n-2)^2$
- Number of cubes with exactly two faces painted = Number of cubes on the edges $= 12(n-2)$
- Number of cubes with exactly three faces painted = Number of cubes on the vertices or corners = 8

CUTTING OF A CUBOID

- When a cuboid is cut into smaller cubes of equal volume, then total number of cubes formed,

$$= \dfrac{(\text{Volume of cuboid})}{(\text{Volume of smaller cube})}$$

$$= \dfrac{(\text{length} \times \text{breadth} \times \text{height}) \text{ of cuboid}}{(\text{side of smaller cube})^3}$$

If a cuboid of dimension $L \times B \times H$ is painted on all sides and is then cut into smaller cubes of dimension $1 \times 1 \times 1$, then

- Number of cubes with zero or no face painted
  = Cubes which are in second layer or inside
  $= (L-2) \times (B-2) \times (H-2)$
- Number of cubes with exactly one face painted
  = Cubes on the surface or face
  $= 2 \times [(L-2) \times (B-2) + (B-2) \times (H-2) + (L-2) \times (H-2)]$
- Number of cubes with exactly two faces painted
  = Number of cubes on the edges $= 4 \times (L + B + H - 6)$
- Number of cubes with exactly three faces painted
  = Number of cubes on the vertices or corners = 8

DICE

When you roll a six-sided dice and add up the number of dots on the opposite faces, if the total is 7, then that dice is called a standard dice.

General dice can have any arrangement of numbers and do not follow the specific sum rule of standard dice. When the sum of digits/dots on opposite faces is not equal to 7, then the dice is called general dice.

Finding Digits/Dots/Words/Letters/Figures/Symbols on opposite faces of a Dice

- **Case 1: Common Digit on Different Faces**
  Let us consider the following two dice positions:
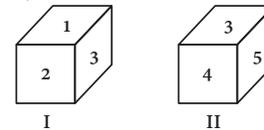
Since the number '3' appears on different faces in two dice positions, we will rotate the dice from the common number i.e., 3 in a clockwise direction to get the following dice position:
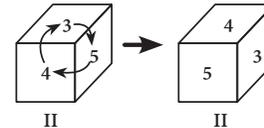
Now comparing (I) with (II)
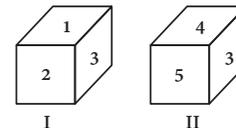
From the above to dice position we can say that 1 is opposite to 4, 2 is opposite to 5 and 3 will be opposite to 6.

- **Case 2: Two Common Digits**
  When two numbers are common in two positions of the dice, the remaining uncommon numbers are opposite to each other.

For example:
If 5 and 6 are common in two positions, and the remaining numbers are 3 and 4, then 3 is opposite to 4.
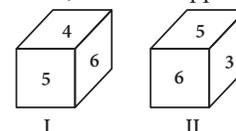
Open Dice

When dice is cut open the alternate places will be opposite to each other
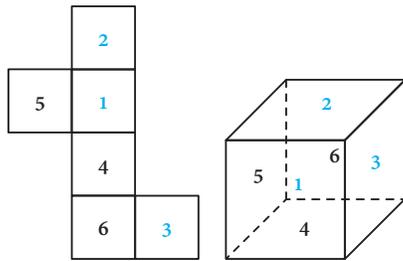
Here the opposites are 2-4, 1-6 and 3-5

Let see methods to solve questions based on these concepts:

1. **If a cube of 12 cm side is divided into smaller cubes of 4 cm side, then**

   (i) **Find the total number of smaller cubes.**

   (*a*) 16   (*b*) 64

   (*c*) 27   (*d*) 32

   Sol: (c)

   n = Side of bigger cube/Side of smaller cube

   n =12/4 = 3

   $n^3 = 3^3 = 27$

   (ii) **Find the total number of corner cubes.**

   (*a*) 16   (*b*) 12

   (*c*) 4   (*d*) 8

   Sol: (d)

   As total number of corner cubes is always 8

   (iii) **What is the total number of middle cubes?**

   (*a*) 16   (*b*) 12

   (*c*) 32   (*d*) 4

   Sol: (b)

   Number of middle cubes = 12(n – 2) = 12(3 – 2) = 12 x 1 = 12

   (iv) **What is the total number of central cubes?**

   (*a*) 6   (*b*) 9

   (*c*) 20   (*d*) 16

   Sol: (a)

   Total number of central cubes = $6 \times (n – 2)^2 = 6 \times (3 – 2)^2 = 6 \times 1^2 = 6$

   (v) **Find the total number of inner central cubes?**

   (*a*) 18   (*b*) 9

   (*c*) 1   (*d*) 81

   Sol: (c)

   Total number of inner central cubes = $(n – 2)^3 = (3 – 2)^3 = 1$

2. **A solid cube of 3 cm side, painted on all its faces, is cut up into small cubes of 1 cm side. How many of the small cubes will have exactly two painted faces?**

   (*a*) 12   (*b*) 8

   (*c*) 6   (*d*) 4

   Sol: Here, n = 3

   Here, edge legth of larger cube = M = 3 cm

   And, edge length of smaller cube = N = 1 cm

   So, $n = \dfrac{M}{N} = \dfrac{3}{1} = 3$

   So, number of cubes with two painted faces

   = $12 \times (n – 2) = 12 \times (3 – 2) = 12$

   Hence, option (a) is correct.

3. **A cuboid of dimensions 7 cm × 5 cm × 3 cm is painted red, green and blue colour on each pair of opposite faces of dimensions 7 cm × 5 cm, 5 cm × 3 cm, 7 cm × 3 cm, respectively. Then the cuboid is cut and separated into various cubes each of side length 1 cm. Which of the following statements is/are correct?**

   I. There are exactly 15 small cubes with no paint on any face.

   II. There are exactly 6 small cubes with exactly two faces, one painted with blue and the other with green.

   Select the correct answer using the code given below:

   (*a*) I Only   (*b*) II Only

   (*c*) Both I and II   (*d*) Neither I nor II

   Sol: The painted surfaces are:
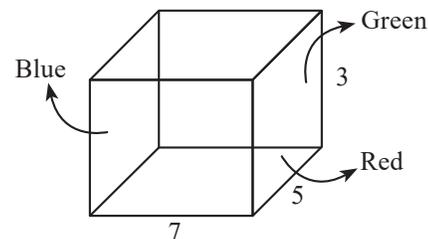
   It is given that the cuboid of 7 cm × 5 cm × 3 cm painted:

   - Two faces of dimensions 7 cm × 5 cm. These faces are painted red.
   - Two faces of dimensions 5 cm × 3 cm. These faces are painted green.
   - Two faces of dimensions 7 cm × 3 cm. These faces are painted blue.

   We have L = 7, B = 5 and H = 3

   Cubes with no painted face:

   So, we need to find out the number of internal cubes in the cuboid.

Number of internal cubes in the cuboid = $(L - 2)(B - 2)$ $(H - 2) = (7 - 2) \times (5 - 2) \times (3 - 2) = 5 \times 3 \times 1 = 15$

So, Statement-I is correct.

Cubes with exactly two faces, one painted with blue and the other with green:

These cubes can be found in the middle of the surface with dimension 7 cm × 3 cm.

So, there will be 4 cubes on each face.

Thus, required number of cubes with one face painted with blue and the other with green = $4 \times 2 = 8$

So, Statement-II is incorrect.

Hence, option (a) is correct.

4. **125 identical cubes are arranged in the form of a cubical block. How many cubes are surrounded by other cubes from each side?** (UPSC CSAT 2023)

(*a*) 27   (*b*) 25

(*c*) 21   (*d*) 18

Sol: We have to find the number of internal cubes, i.e. the number of cubes that are not exposed.

There are 125 cubes.

Since, $5^3 = 125$

So, $n = 5$ i.e. we have 5 identical cubes on each edge.

Number of internal cubes = $(n - 2)^3 = (5 - 2)^3 = 27$

Hence, option (a) is correct.

5. **A cube has six numbers marked 1, 2, 3, 4, 5 and 6 on its faces. Three views of the cube are shown below:**
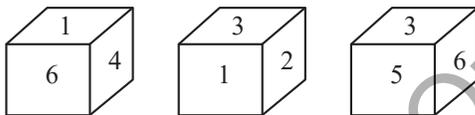
What possible numbers can exist on the two faces marked Ⓐ and Ⓑ respectively on the cube?

(UPSC CSAT 2013)
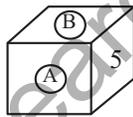
(*a*) 2 and 3   (*b*) 6 and 1

(*c*) 1 and 4   (*d*) 3 and 1

Sol: Observe the 1st figure: 4 and 6 are adjacent to 1.
In the 2nd figure, 2 and 3 are adjacent to 1.

Numbers which are adjacent to 1 are: 2, 3, 4, and 6

So, the number which is opposite to 1 is 5 and vice versa.

Hence, 1 can not be the value of A and B.

So, options (b), (c), and (d) are eliminated.

Hence, option (a) is correct.

6. **Six squares are colored, front and back, red (R), blue (B), yellow (Y), green (G), white (W), and orange (O), and are hinged together as shown in the figure given below. If they were folded to form a cube what would be the face opposite to the white face?** (UPSC CSAT 2012)
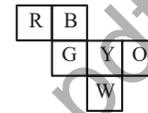
(*a*) R   (*b*) G

(*c*) B   (*d*) O

Sol: We have been given a cube. Suppose Y forms the top of the cube. As we start folding the cube, we will find that the faces with G, B, O, and W are all adjacent to the Y face.

The face labeled R will end up at the bottom of the cube. This means that R and Y are on opposite sides of the cube.

Similarly, G is opposite to O, and W is opposite to B as shown in figure:
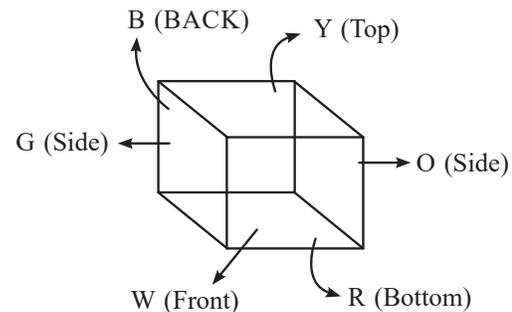
So, the face with B (blue) is directly across from the face with W (white).

Hence, option (c) is correct.

❖   ❖   ❖

TIPS FOR SOLVING BLOOD RELATION PROBLEMS

- Draw Family Tree: Visual representation can simplify complex relationships.
- Use '+' for male members and '-' for female members in the family tree diagram.
- Start with Direct Relations: Identify and mark direct relations first, then move to indirect ones.
- Use a continuous horizontal line to connect siblings.
- Draw parallel lines between a husband and wife to indicate marriage.

Example of a Family Tree

$$A(+) == B(-)$$
$$D(-) \text{---} C(+) == E(-)$$
$$H(-) \text{---} F(+) \text{---} G(+)$$
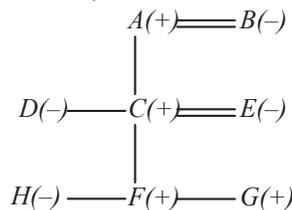
- (*A*, *B*) and (*C*, *E*) are husband and wife pairs.
- (*C*, *D*) and (*F*, *H*, *G*) are sibling groups.
- *A*, *C*, *F*, *G* are male members of the family.
- *B*, *D*, *E*, *H* are female members of the family.
- (*A*, *B*) are 1st generation, (*C*, *D*, *E*) are 2nd generation and (*F*, *G*, *H*) are the 3rd generation.

Let see methods to solve questions based on these concepts:

1. **A family of two generations consisting of six members '*P*', '*Q*', '*R*', '*S*', '*T*' and '*U*' has three males and three females. There are two married couples and two unmarried siblings. '*U*' is *P*'s daughter and '*Q*' is *R*'s mother-in-law. '*T*' is an unmarried male and '*S*' is a male. Which one of the following is correct?**

(UPSC CSAT 2020)

(*a*) *R* is *U*'s husband   (*b*) *R* is *S*'s wife

(*c*) *S* is unmarried   (*d*) None of the above

Sol: It is given that

There are two generations.

There are 6 members – 3 males and 3 females

Male Members are: '*T*', '*S*' and one more

Female members are : '*U*', '*Q*' and one more

We do not know about the gender of '*P*' and '*R*'.

There are two couples and the rest of the two are unmarried siblings.

'*P*', '*Q*' and '*R*' are definitely married because it is given that '*U*' is *P*'s daughter and '*Q*' is *R*'s mother-in-law.

'*T*' is single (un-married).

We do not know whether '*S*' and '*U*' are married or not.

As '*U*' is *P*'s daughter and '*Q*' is *R*'s mother-in-law, it means that '*P*' and '*Q*' belong to the senior generation and '*U*' and '*R*' belong to the junior generation.

For making the family diagram we will use the following notations:

'+' for male members and '−' for female members

A continuous horizontal line to connect siblings.

Parallel lines between a husband and wife to indicate marriage.

Vertical line to connect parents to their children.

So, we get the following family diagram:

$$Q(-) == P(+)$$
$$R(-) == S(+) \text{---} U(-) \text{---} T(+)$$
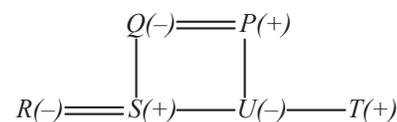
'*U*' and '*T*' are the unmarried siblings.

And, '*R*' is *S*'s wife.

Hence, option (b) is correct.

2. **A joint family consists of seven members A, B, C, D, E, F, and G with three females. G is a widow and sister-in-law of D's father F. B and D are siblings and A is daughter of B. C is cousin of B. E is mother of B. Who is E?** **(UPSC CSAT 2019)**

1. Wife of F
2. Grandmother of A
3. Aunt of C

Select the correct answer using the code given below:

(*a*) 1 and 2 only   (*b*) 2 and 3 only

(*c*) 1 and 3 only   (*d*) 1, 2 and 3

Sol: '+' is used for male members and '–' is used for female members

A continuous horizontal line to connect siblings.

Parallel lines between a husband and wife to indicate marriage.

Vertical line to connect parents to their children.
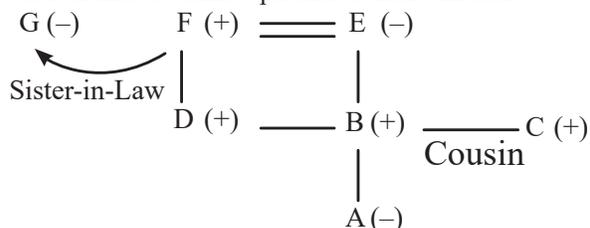
From above family diagram we can say that:

'E' is wife of 'F', 'E' is grandmother of 'A' and 'E' is aunt of 'C'

Hence, option (d) is correct.

3. **Consider the following relationships among members of a family of six persons 'A', 'B', 'C', 'D', 'E' and 'F':**

    I. The number of males equals that of females.

    II. 'A' and 'E' are sons of 'F'.

    III. 'D' is the mother of two, one boy and one girl.

    IV. 'B' is the son of 'A'.

    V. There is only one married couple in the family at present.

    Which one of the following inferences can be drawn from above? (UPSC CSAT 2017)

(a) 'A', 'B' and 'C' are all females.

(b) 'A' is the husband of 'D'.

(c) 'E' and 'F' are children of 'D'.

(d) 'D' is the daughter of 'F'.

**Sol:** For making the family diagram we will use the following notations:

'+' for male members and '–' for female members

A continuous horizontal line to connect siblings.

Parallel lines between a husband and wife to indicate marriage.

Vertical line to connect parents to their children.
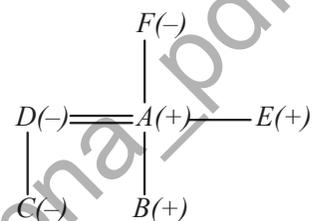
'A' and 'D' are couples. They have a daughter 'C' and a son 'B'.

'F' is the mother of 'A' and 'E' and both 'A' and 'E' are male.

So in total there are three males 'A', 'B' and 'E' and three females 'D', 'C' and 'F'.

So, we can say that 'A' is the husband of 'D'.

Hence, option (b) is correct.

Reading Comprehension

Reading Comprehension Basics

HOW TO APPROACH READING COMPREHENSION IN CSAT ?

Solving Reading comprehension in the CSAT can be challenging, but here are some tips that you can use to improve your performance.

- **Improve your reading speed and comprehension skills:** Reading is a skill that you can improve with practice. Try to read as much as possible and pay attention to the structure of the passages. Focus on understanding the main idea and the supporting details.

- **Highlight or underline important information:** When you come across important information, such as dates, names, and key concepts, highlight or underline them. This will help you to find them quickly when you need to answer the questions.

- **Identify the type of question being asked:** There are different types of questions that can be asked, such as factual questions, inference questions, and vocabulary questions. Understanding the type of question being asked will help you to approach the question correctly.

- **Practise previous year question papers:** Practising previous year question papers will give you an idea of the type of questions that can be asked in the exam. This will also help you to develop your time management skills.

- **Look for context clues:** Sometimes, unfamiliar words or phrases may appear in the passage. If you come across such words, look for context clues such as synonyms or antonyms, to help you understand the meaning of the word or phrase.

- **Focus on the structure of the passage:** The structure of the passage can give you important clues about the author's purpose and main ideas. Look for transition words, such as "however" and "therefore," to understand the relationships between different ideas.

- **Make educated guesses:** If you are unsure about an answer, try to make an educated guess by eliminating obviously incorrect answers and making an educated guess based on the information you have.

- **Practice active reading:** When reading a passage, try to actively engage with the text by asking yourself questions and making connections to your own experiences and knowledge. This will help you to understand the passage better and retain the information more effectively.

Manage your time wisely: Time management is crucial in the UPSC CSE CSAT exam. Make sure to allocate your time wisely, and don't spend too much time on any one question. If you get stuck on a question, move on and come back to it later if you have time.

Apart from these tips, as a matter of last resort; elimination techniques, as discussed below, also help candidates to arrive at the correct answer in a limited time period.

OPTIONS ELIMINATION IN READING COMPREHENSION

- In any exam which tests your reading comprehension, including UPSC-CSAT, there are five primary criteria which helps you decide that a particular option is incorrect.

- Often, it is observed in UPSC-CSAT that a candidate is able to eliminate two options comfortably. But UPSC gives very close choices for the remaining two options which makes it really difficult to eliminate the third option to arrive at the answer, in a limited time frame.

- In such situations having predefined criteria as to in which all ways an option can be incorrect will give candidates much needed clarity of thought and ultimately increase their speed in solving reading comprehension based questions.

- However, candidates should keep in mind that first they should always attempt to arrive at the right answer without thinking about wrong options. If that does not work, then only one should resort to elimination techniques.

Five Criteria to Eliminate Options

1. Out of the Scope of Passage

This refers to options, which are not even mentioned in the passage explicitly or implicitly, that is, you find no direct or indirect reference of the option in the passage.

Let us consider the following example to understand this:

Passage

Humans often seek change due to a combination of innate curiosity, a desire for improvement, and a coping mechanism for discomfort or dissatisfaction. Evolutionarily, adaptability has been crucial for survival, driving the inclination to modify circumstances. In this ever-evolving journey of life, the

Japanese philosophy of Uketamo emerges as a beacon of light. Uketamo, translated loosely as "to accept." The philosophy teaches that all things, whether positive or challenging, are fleeting. It encourages individuals to acknowledge and receive both the joys and sorrows of life without resistance, fostering a profound sense of equilibrium.

Q. Which of the following statements most accurately represents the central idea of the passage?

(*a*) Changes in human lives are often desirable.

(*b*) Changes should be avoided to bring about the sense of equilibrium.

(*c*) Acceptance towards life's ups and downs can lead to peace.

(*d*) Depression is a modern day phenomenon, which was not even there in the past.

Sol: (*c*)

Here, one can easily eliminate option (d) since it is out of the scope of the passage. Passage talks about change and how Uketamo helps individuals to deal with the changes. While option (d) talks about depression, which finds no direct or indirect mention in the passage, hence this option is out of the scope of the passage.

2. Extreme Options

This refers to an option which has been arrived at after making extreme assumptions or very long inferences with respect to the content given in the passage. To put it in simpler terms too, one needs to apply way too many logics in addition to the information given in the passage to see this particular option as true.

Let us consider following example to understand this:

Passage

Recent advancements in artificial intelligence (AI) have led to significant progress in the field of renewable energy. AI algorithms are helping researchers design more efficient solar panels, predict wind patterns for better turbine placement, and develop smart grids that optimize energy distribution. While the potential of AI for tackling climate change is undeniable, some experts warn that its increasing dependence on fossil fuels for data processing and hardware production could ultimately undermine its environmental benefits.

Q. Based on the above passage, which of the following assumption(s) is/are valid?

(*a*) AI will ultimately enable renewable technology to replace fossil fuels completely in the coming future.

(*b*) Data processing requires extensive computer infrastructure powered by fossil fuels.

(*c*) Manufacturing of hardware components for AI systems heavily pollutes the environment.

(*d*) Both (*b*) & (*c*)

Sol: (*d*)

Here, option (a) can be eliminated since it makes an extreme assumption. At First, the extreme assumption is that AI will enable renewable energy to replace fossil fuels. We cannot conclude this based on the information given in the passage. Passage only talks about AI making renewable energy technology more efficient. Secondly, "Way too long" logic is applied here in assuming whether renewable energy will replace fossil fuels or not. This is nowhere explicitly mentioned in the passage. Hence, this option is an extreme option.

3. Partially Correct

This refers to options which may appear true at first place, but only a part of the option, say a word or two makes this option incorrect. Hence to eliminate such options a careful, word to word reading is required. Part, which makes this option incorrect, can be an extreme word or something not mentioned in the passage even implicitly.

Let us consider following example to understand this:

Passage

In the landmark Keshavananda Bharti Case, the Basic Structure Doctrine emerged as a judicial doctrine safeguarding the core principles of the Constitution. The judiciary asserted that while Parliament has the power to amend the Constitution, it cannot alter its basic structure. This doctrine, articulated by the Supreme Court of India, ensures the preservation of essential features such as democracy, rule of law, and individual liberties. Keshavananda Bharti's case marked a pivotal moment in constitutional jurisprudence, establishing a framework that guards against arbitrary amendments threatening the foundational integrity of the Constitution."

Q. Which of the following assumptions is implicit in the passage?

(*a*) The Basic Structure Doctrine may require periodic review to adapt to evolving constitutional principles.

(*b*) In the landmark Keshavananda Bharti Case, the Basic Structure Doctrine emerged as the only judicial doctrine safeguarding the core principles of the Constitution.

(*c*) The preservation of essential features like democracy and individual liberties is a universally uncontested aspect of constitutional jurisprudence.

(*d*) The Basic Structure Doctrine, as articulated in the Keshavananda Bharti Case, is immune to legal challenges.

Sol: (*a*)

Here, we can eliminate option (b) since it is a rotten fruit. At first it appears correct since it is explicitly mentioned in the opening remarks of the passage. But only one word 'only' makes this option an incorrect one.

Based on the information given in the passage, we can not infer whether it is the 'only' judicial doctrine safeguarding core principles of the constitution or not. There may be other such doctrines as well.

4. Contextually Wrong Options

This refers to options which are true in itself based on the information in the passage, when examined independently. But they do not specifically answer the question. For instance, the question is asking the underlying tone of the passage, but the option simply states one of the facts mentioned in the passage and not the underlying tone.

Let us consider following example to understand this:

Passage

Philosophers through the ages have constantly reminded us of this underlying universal principle that we are a small but integral part of this web of life. "That which isn't good for the hive, isn't good for the bee," said Marcus Aurelius. Just as each organ in our body has its own individual function, but always towards the wellbeing of the whole; each of us has a role, and any action that is not in the benefit of the collective, ultimately cannot benefit the individual. Only when we truly learn to recognise that we are not separate from nature but a part of this one life, can we positively alter the way we consume.

Q. In the context of the passage, what is the most logical inference that can be drawn?

(*a*) Recent emphasis on individual rights goes against the idea of collective benefits.

(*b*) Collective benefits should be placed over individual benefits.

(*c*) Each one of us has a role which ultimately leads to the benefit of the collective.

(*d*) We are separate from nature but we must act in sync with nature.

Sol: (*b*)

Here, at first instance option (c) may appear correct since it is explicitly mentioned in the passage. But upon careful reading of the question we recognize that the question is asking for the most logical inference.

Option (c) is a straight fact stated based on information given in the passage. Inference is something which we derive based on logical and rational reasoning and is not mentioned directly in the passage. Hence, option (c) is contextually wrong.

5. True in Real World but Incorrect as per passage

This refers to options which are true in real world situations and normal circumstances. These options are true based on common sense and general knowledge. But based on the information given in the passage and the question that follows, these options are wrong. Hence one needs to be always aware of the fact that answers need to be based on the passage solely in order to eliminate such options.

Let us consider following example to understand this:

Passage

As India's Central Electricity Authority (CEA) marks its 50th year, its vital role in shaping the nation's energy backbone shines brightly. From fostering grid expansion to ensuring grid stability, the CEA has facilitated affordable electricity access for millions. Its technical expertise continues to be pivotal, even as the landscape shifts towards renewables and climate concerns. Balancing these new demands with its traditional strengths will be crucial for the CEA's future impact.

Q. What is the most logical and crucial message conveyed by the passage?

(*a*) CEA is a statutory organisation constituted under Electricity Supply Act, 1948

(*b*) Transitioning to renewables necessitates dismantling the CEA's traditional framework and starting anew.

(*c*) Balancing affordability, renewable integration, and climate goals will define the CEA's future effectiveness.

(*d*) CEA advises government on policy matters and formulates plans for development of electricity systems

Sol: (*c*)

Here, if we rely on pre-existing general knowledge that we have based on our study of GS subjects, options (a) and (d) are true. But based on the information given in the passage and the question that follows, we simply can not determine whether these options are correct or not.

While attempting the CSAT, in a limited time frame, in a hurry it is a possibility that one marks one of these options as correct. Especially if you are just skimming through the passage and not reading both passage and the question carefully.

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1 Inference Based Reading Comprehension

INTRODUCTION

- These questions necessitate candidates to extend their understanding beyond what is explicitly mentioned in the text, delving into the realm of implicit meanings and potential consequences.
- The essence of inference-based questions lies in the capacity to make informed interpretations and extrapolations based on the presented information. Rather than relying solely on explicitly stated facts, candidates must engage in a deeper exploration of the passage to identify subtle cues, implicit relationships, and nuanced connections.
- Candidates are tasked with making educated guesses and drawing reasonable inferences, showcasing their ability to derive meaningful insights.

ASSOCIATED KEYWORDS

- The associated keywords with these types of questions are "most logical and rational inference", "most critical inference". The "inference" etymologically means "carry forward". It is related to the steps in reasoning that allows one to move from premises to logical consequences.

- The most appropriate option would be directly related to the information given explicitly in the passage through logical reasoning. Understanding of this logical link between the messages in the passage and the options will guide you to reach the right answer.

NATURE OF INFERENCE BASED QUESTIONS

- You can recognize these questions by looking for the word "inference". The options you get are like logical next steps based on what's said in the passage. To answer them correctly, it's crucial to pick the choice that you can directly figure out from the passage and that stays within what the passage talks about.
- When dealing with inference questions, you need to go a bit beyond just the obvious details. You have to think about what makes sense from the information given in the passage. Don't choose options that talk about things not mentioned in the passage. Stick to what's in the passage.
- The main rule for all kinds of questions is to stay within what the passage talks about. So, when you're choosing your answer, make sure it fits with what's written in the passage. This way, you show that you understand the information well and can make logical conclusions based on what you read.

Tips and Tricks:
- When you spot the inference related keyword, avoid quickly scanning through the passage. Take your time to read the passage carefully and fully understand it.
- Identify and underline keywords in the passage, particularly phrases that capture the central theme. Inferences are based on the 'step forward' of this central theme.
- Focus on the concluding remarks of the passage. When confused between 2 options related to inference mark the one that aligns with the concluding remarks.
- The most suitable option is the one that reflects the content of the whole passage whereas other confusing options will conclude a part of the passage leaving other parts unaddressed. Look for such options.

PREVIOUS YEAR QUESTIONS

Directions for the following items: Read each of the following passages and answer the items that follow. Your answers of these items should be based on the passages only.

Passage

Good corporate governance structures encourage companies to provide accountability and control. A fundamental reason why corporate governance has moved onto the economic and political agenda worldwide has been the rapid growth in international capital markets. Effective corporate governance enhances access to external financing by firms, leading to greater investment, higher growth and employment. Investors look to place their funds where the standards of disclosure, of timely and accurate financial reporting, and of equal treatment to all stakeholders are met. **(UPSC CSAT 2023)**

1. **Which of the following statements best reflects the logical inference from the passage given above?**

   (*a*) It is an important agenda of the countries around the world to ensure access to good external financing.

   (*b*) Good corporate governance improves the credibility of the firms.

   (*c*) International capital markets ensure that the firms maintain good corporate governance.

   (*d*) Good corporate governance paves the way for robust supply chains.

Sol: **Option (*a*) is incorrect:** Though the passage talks about corporate governance moving onto a global economic and political agenda, it is not the most logical inference since the central idea of the passage is how good corporate governance benefits corporations and ultimately countries around the world.

Option (*c*) is incorrect: The passage mentions how international capital market dynamics has brought the idea of corporate governance to the forefront. However, it is just one fundamental reason due to which the idea of corporate governance has gained importance. To infer that it would 'ensure' corporate governance would be a stretch. Further, the central idea of the passage is rather opposite of this i.e. how effective corporate governance enhances access to the international capital market.

Option (*d*) is incorrect as the passage does not, explicitly or implicitly, talk about supply chain.

Option (*b*) is correct as the passage mentions that 'Good corporate governance structures encourage companies to provide accountability and control' and 'Investors look to place their funds where the standards of disclosure, of timely and accurate financial reporting, and of equal treatment to all stakeholders are met.' These two statements together infer that good corporate governance improves credibility of the firms.

Hence, option (b) is correct.

Passage

The demographic dividend, which has begun in India and is expected to last another few decades, is a great window of opportunity. The demographic dividend is basically a swelling in the working age population, which conversely means that the relative ratio of very young and very old will, for a while, be on the decline. From the experience of Ireland and China, we know that this can be a source of energy and an engine of economic growth. The demographic dividend tends to raise a nation's savings rate since in any nation, it is the working age population that is the main saver. And since the savings rate is an important driver of growth, this should help elevate our growth rate. However, the benefits of demographic dividend depend on the quality of the working age population. And this implies bringing back the importance of education, acquisition of skills and human capital. **(UPSC CSAT 2022)**

2. **With reference to the passage, which of the following inferences can be drawn?**

   I. Demographic dividend is an essential condition for a country to rapidly increase its economic growth rate.

   II. Promotion of higher education is an essential condition for a country for its rapid economic growth.

   Select the correct answer using the code given below.

   (*a*) I only
   (*b*) II only
   (*c*) Both I and II
   (*d*) Neither I nor II

Sol: **Statement-I is incorrect:** The demographic dividend is an essential condition but we can not conclude from the passage that it will rapidly increase the country's economic growth rate.The increase could be of slow pace as well.

Statement-II is incorrect: From the given information in the passage it can not be inferred explicitly or implicitly that higher education is an essential condition for a country's rapid economic growth.

Hence, option (d) is correct.

Passage

Researchers simulated street lighting on artificial grassland plots containing pea-aphids, sap-sucking insects, at night. These were exposed to two different types of light a white light similar to newer commercial LED lights and an amber light similar to sodium street lamps. The low intensity amber light was shown to inhibit, rather than induce, flowering in a wild plant of the pea family which is a source of food for the pea-aphids in grasslands. The number of aphids was also significantly suppressed under the light treatment due to the limited amount of food available. **(UPSC CSAT 2021)**

3. **Which one of the following statements best reflects the most critical inference that can be made from the passage given above?**

   (*a*) Low intensity light has more adverse effects on the plants as compared to high intensity light.

   (*b*) Light pollution can have a permanent adverse impact on an ecosystem.

   (*c*) White light is better for the flowering of plants as compared to the light of other colours.

   (*d*) Proper intensity of light in an ecosystem is important not only for plants but for animals too.

Sol: **Option (*a*) is incorrect:** The passage does not contain information about the effect of high intensity light. Moreover the statement given in option A is not the most crucial inference which can be drawn from the passage.

Option (*c*) is incorrect as nowhere in the passage, it is mentioned that white light is superior to other colours for plants flowering.

Option (*d*) is incorrect: The passage doesn't have enough information from which we can infer what effect white light will have on animals.

Option (*b*) is correct and it can be easily inferred from the passage. Since the concluding remarks of the passage suggests that the number of aphids can be significantly suppressed under light treatment. Also the passage talks about other effects of light on plants.

Hence, option (b) is correct.

ASSOCIATED KEYWORDS

- "Most logical, rational & critical message/best reflect the passage" are common keyword used in these kinds of questions.

- **The Most Critical, Rational, and Logical Message:** This engages your capacity for analysis. You are evaluating the author's position and choosing the one that, when supported by logic and evidence, offers the most sensible, objective conclusion.

- **Best Reflect Tone:** This focuses on how the author feels about the topic. You are choosing the response that most accurately expresses their emotional state, whether it be objective, subjective, formal, informal, serious, or lighthearted.

NATURE OF MESSAGE BASED QUESTIONS

- The "most logical message," "best reflects tone," "crux," and "central idea" questions are all fundamentally searches for the passage's synopsis. They pose the question, "Which option best captures the essence of the entire passage?" to put it simply.

- Consider the passage as a complex fabric which is made up of different threads representing arguments, details, and opinions the author wants to discuss in the passage. The goal of these questions is for you to pinpoint the one central thread that unites the entire fabric. The main idea and overall message of the author are summed up in this central thread.

- The right answer isn't about picking a side, despite the fact that options may entice you with their implications or solutions. It's about realising the overarching idea that runs through the whole chapter.

Tips and Tricks:

Eliminate options with extreme language, factual inaccuracies, or emotional appeals. Pay close attention to transitions and how the author moves from evidence to conclusions. Look for words that convey emotion or opinion. Consider the title and introductory/concluding statements. Don't get bogged down by specific details or supporting arguments

PREVIOUS YEAR QUESTIONS

Directions for the following items: Read each of the following passages and answer the items that follow. Your answers of these items should be based on the passages only.

Passage

In India, a majority of farmers are marginal and small, less educated and possess low adaptive capabilities to climate change, perhaps because of credit and other constraints. So, one cannot expect autonomous adaptation to climate change. Even if it was possible, it would not be sufficient to offset losses from climate change. To deal with this, adaptation to climate change is paramount, alongside a fast mitigation response. Another solution is to have a planned or policy-driven adaptation, which would require the government to come up with policy recommendations. Perception is a necessary prerequisite for adaptation. Whether farmers are adapting agricultural practices to climate change depends on whether they perceive it or not. However, this is not always enough for adaptation. It is important how a farmer perceives the risks associated with climate change. **(UPSC CSAT 2023)**

1. **Which one of the following statements best reflects the most logical and rational message conveyed by the author of the passage?**

   (*a*) Adaptation to climate change and mitigation response are basically the responsibilities of the government.

   (*b*) Climate change causes a change in government policies regarding land use patterns in the country.

   (*c*) Risk perceptions of farmers are important for motivating them to take adaptation decisions.

   (*d*) Since mitigation is not possible, governments should come up with policies for quick response to climate change.

Sol: Option (*a*) is incorrect: The passage recognises farmers' limitations owing to their lack of resources and knowledge, even as it stresses the necessity of government intervention in planned and policy driven adaptation. But perception among farmers is also a critical factor as per the passage. Thus, there are roles for the government and farmers alike.

Option (b) is incorrect: There is no clear connection made in the passage between changes in land use patterns and climate change. Also the phrase 'land use pattern' is beyond the scope of the passage.

Option (d) is incorrect: It is not stated in the passage that mitigation is not possible. Adaptation along with fast mitigation response, is the solution prescribed in the passage.. Furthermore, the author supports planned and policy-driven adaptation rather than merely prompt responses.

Option (c) is correct: This choice is in line with the passage's emphasis on how crucial it is for farmers to perceive the risks in order to be motivated to take adaptive action. Perception is emphasised by the author as a "necessary prerequisite" for adaptation.

Hence, option (c) is correct.

Passage

"In simple matters like shoe-making, we think only a specially trained person will serve our purpose, but in politics, we presume that everyone who knows how to get votes knows how to administer a State. When we arc ill, we call for a trained physician, whose degree is a guarantee of specific preparation and technical competence--wc do not ask for the handsomest physician, or the most eloquent one : well then, when the whole State is ill should we not look for the service and guidance of the wisest and the best?"

(UPSC CSAT 2022)

2. **Which one of the following statements best reflects the message of the author of the passage?**

   (a) We assume that in a democracy, any politician is qualified to administer a State.

   (b) Politicians should be selected from those trained in administration.

   (c) We need to devise a method of barring incompetence from public office.

   (d) As voters select their administrators, the eligibility of politicians to administer a State cannot be questioned.

Sol: **Option (a) is incorrect:** It contradicts what the passage is trying to convey.

Option (b) is incorrect as the passage mentions that those who are wise, should work in administration and does not mention anything about training in administration.

Option (d) is incorrect since it is contradictory to what the passage is trying to convey in "when the whole State is ill should we not look for the service and guidance of the wisest and the best?"

Option (c) is correct since it best summarises the meaning of the passage which is reflected in concluding remarks of the passage.

Hence, option (c) is correct.

Passage

India faces a challenging immediate future in energy and climate policy-making. The problems are multiple: sputtering fossil fuel production capabilities; limited access to electricity and modern cooking fuel for the poorest; rising fuel imports in an unstable global energy context; continued electricity pricing and governance challenges leading to its costly deficits or surplus supply; and not least, growing environmental contestation around land, water and air. But all is not bleak: growing energy efficiency programmes; integrated urbanisation and transport policy discussion; inroads to enhancing energy access and security; and bold renewable energy initiatives, even if not fully conceptualised, suggest the promise of transformation. **(UPSC CSAT 2021)**

3. **Which one of the following statements best reflects the critical message conveyed by the passage given above?**

   (a) India's energy decision-making process is more complex and is ever interconnected.

   (b) India's energy and climate policy is heavily tuned to sustainable development goals.

   (c) India's energy and climate actions are not compatible with its broader social, economic and environmental goals.

   (d) India's energy decision-making process is straightforward supply-oriented and ignores the demand side.

Sol: **Option (b) is incorrect:** India's energy decision making policy is yet not heavily tuned towards sustainable development goals which is evident from various challenges discussed in the passage.

Option (c) is incorrect: Since we do not have enough information in the passage regarding what are India's broader social, economical and environmental goals.

Option (d) is incorrect: The passage does not discuss implicitly or explicitly about supply and demand side of India's energy decision making process.

Option (a) is correct: Message conveyed in this option is evident from various multidimensional challenges and opportunities discussed in the passage.

Hence, option (a) is correct.

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ASSOCIATED KEYWORDS

"Crux of the passage/central idea/Underlying tone of the passage" is a common keyword used in these kinds of questions. These questions go deeper in the very heart of the text and demands a deeper insight into the author's argument than a surface level understanding.

NATURE OF CRUX BASED QUESTIONS

● See the passage as an intricate machinery. The one thing that all of its functioning depends on is the crux. It is the central claim, the central idea around which all other arguments are centred. This tests your ability to pinpoint the single most important point the author wants to convey. It could be mentioned clearly or subtly.

● Think of the passage as a map. The destination, or overarching theme that directs the reader's journey through the text's various points, is the central idea. Like the crux, this concentrates on the central idea or primary theme of the passage. All the components come together to form the overall message.

Tips and Tricks:

Before delving into specifics, quickly scan the text to acquire a basic understanding of its organisation and key ideas. This first paragraph usually offers a useful foundation for understanding details. Instead of making assumptions, base your responses on specific examples from the text.

PREVIOUS YEAR QUESTIONS

Directions for the following items: Read each of the following passages and answer the items that follow. Your answers of these items should be based on the passages only.

Passage

Many people are not eating the right food. For some, it is simply a decision to stick with food they enjoy but which is not too healthy. This is leading to an increase in non-communicable diseases. This in turn leads to a major burden on our health-care systems that have the potential to derail the economic progress which is essential for the poor to improve their lives. For others, it is about limited access to nutritious food or a lack of affordability, leading to monotonous diets that do not provide the daily nutrients for them to develop fully. Part of the reason nutrition is under threat worldwide is that our food systems are not properly responding to nutritional needs. Somewhere along that long road from farm to fork, there are serious detours taking place. **(UPSC CSAT 2023)**

1. **Which one of the following statements best reflects the crux of the passage?**

   (*a*) The scheme of Universal Basic Income should be implemented worldwide as a way of poverty alleviation.

   (*b*) We must place food-based nutrition at the centre of our policy debate.

   (*c*) Nutritional status of food should be improved by creating appropriate genetically modified crops.

   (*d*) Using modern food processing technologies, we must fortify food items with required nutrient elements.

Sol: **Option (*a*) is incorrect** because the passage concentrates on more general nutritional issues rather than just poverty alleviation, even though poverty can restrict access to nourishing food. It highlights poor dietary choices, barriers to access, and weaknesses in food systems, pointing to the necessity for all-encompassing policy solutions that go beyond UBI.

Option (*c*) is incorrect because the passage doesn't expressly support genetic modification, even though it might be able to improve some nutritional aspects of food. It critiques not just the kinds of crops grown but the entire food chain. It is recommended that policy concentrate more on nutrition.

Option (d) is incorrect: Although the passage points out shortcomings and diversion in the entire food system from farm to fork, fortification can be a beneficial intervention. It implies that systemic changes are required throughout the food chain and goes beyond simply adding nutrients to processed foods.

Option (b) is correct as directly supports the main argument of the passage, it is correct. It highlights how crucial it is to give nutrition top priority and make it a major talking point when it comes to policy. The passage outlines various obstacles that impact nutrition and suggests that systemic solutions are necessary.

Hence, option (b) is correct.

Passage

With steady economic growth, higher literacy and increasing skill levels, the number of Indian middle-class families has gone up exponentially. Direct results of the affluence have been changes in dietary patterns and energy consumption levels. People have moved to a higher protein-based diet like milk products, fish and meat, all of which need significantly more water to produce than cereal based diets. Increasing use of electronic and electric machines/gadgets and motor vehicles needs more and more energy and generation of energy needs water. **(UPSC CSAT 2022)**

2. **Which one of the following statements best reflects the crux of the passage?**

   (*a*) People should be persuaded to continue with the mainly Indian traditional cereal-based diets.

   (*b*) India needs to focus on developing agricultural productivity and capacity for more energy generation in the coming years.

   (*c*) Modern technological developments result in the change of cultural and social behaviour of the people.

   (*d*) Water management practices in India need to change dramatically in the coming years.

Sol: **Option (*a*) is incorrect:** This option is beyond the scope of the passage. The passage has no recommendations with regards to dietary choices of Indian people.

Option (*b*) is incorrect: This option provides an incorrect solution. Passage focuses on water and not on agricultural production and energy generation.

Option (*c*) is incorrect: Focus of the passage is water management rather than cultural and social behaviour.

Option (d) is correct: "All of which need significantly more water to produce than cereal based diets. Increasing use of electronic and electric machines/gadgets and motor vehicles needs more and more energy and generation of energy needs water:" based on this sentence, option (*d*) best reflects crux of the passage.

Hence, option (d) is correct.

Passage

A study conducted on the impacts of climate change over the Cauvery basin of Tamil Nadu using regional climate models showed an increasing trend for maximum and minimum temperatures, and a decrease in the number of rainy days. These climatic shifts will have an impact on the hydrological cycles of the region, lead to more run-off and less recharge, and affect the groundwater tables. Further, there has been an increase in the frequency of droughts in the State. This has driven farmers to increase dependency on groundwater resources to secure their crops. **(UPSC CSAT 2021)**

3. **Which one of the following statements best reflects the crux of the passage given above?**

   (*a*) Development of regional climate models helps in choosing climate-smart agricultural practices.

   (*b*) Heavy dependence on groundwater resources can be reduced by adopting dry-land cropping systems.

   (*c*) Climate changes increase the criticality of water resources while simultaneously threatening it.

   (*d*) Climate changes cause the farmers to adopt unsustainable livelihoods and risky coping strategies.

Sol: **Option (*a*) is incorrect** as it is nowhere mentioned in the passage and so it cannot reflect the crux of the passage.

Option (*b*) is incorrect as it is also not mentioned anywhere in the passage and thus remains an incorrect crux.

Option (*d*) is incorrect as it is nowhere mentioned and the cause of climate change might not cause farmers to adopt newer strategies.

Option (*c*) is correct as the concluding remarks direct the passage towards this crux of climate change critically impacting the water resources.

Hence, option (c) is correct.

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4 Implication Based Reading Comprehension

INTRODUCTION

- Questions that focus on implications assess candidates' skills in comprehending the broader consequences, expected outcomes, impacts on stakeholders, unintended ramifications, and suggested results arising from the information provided in a passage. These inquiries delve deeper than the surface-level understanding and demand candidates to decipher the possible effects or implications embedded in the ideas presented.
- Implication-based questions necessitate candidates to move beyond the apparent meaning of the text, encouraging them to draw conclusions and make connections between the information provided and its potential repercussions.
- Candidates are not only required to grasp the explicit details but also to discern implicit suggestions and consequences seamlessly woven into the passage that demonstrate the candidate's proficiency in thinking critically and foreseeing the broader implications of the information at hand.

ASSOCIATED KEYWORDS

- The crucial keywords in these questions include 'rational/ practical/plausible implication,' 'implied from the passage,' and 'what does the author imply.'
- While inference and implication share similar meanings, inference is derived through logical reasoning by the message receiver. In contrast, implication refers to the author's intended future conclusion in their message.

NATURE OF IMPLICATION BASED QUESTIONS

- You can recognize these questions by looking for the word "implication" or "implied". The question generally talks about a central message and would tend to ask the future consequences around the central theme.
- It's important to identify the prospective message the author aims to convey. The ideal answer should relate to an expected future development, outcome, effect, or ramification within the passage's boundaries.

Tips and Tricks:

- When you spot the implication related keyword, avoid quickly scanning through the passage. Take your time to read the passage carefully and fully understand it.
- Identify and underline keywords in the passage, particularly phrases that capture the central theme. Implications are based on the future consequences of this central theme.
- Focus on the concluding remarks of the passage. When confused between 2 options related to implication mark the one that aligns with the concluding remarks.
- The most suitable option is the one that talks about the future consequences or upcoming ramifications, confusing options may just paraphrase the information given explicitly in the passage. The information of the passage directly can not be an implication.

PREVIOUS YEAR QUESTIONS

Directions for the following items: Read each of the following passages and answer the items that follow. Your answers of these items should be based on the passages only.

Passage

More than half of Indian women and almost a quarter of Indian men of working age suffer from anaemia. According to studies, they are anywhere from 5-15% less productive than they could be, as a result thereof. India also has the largest tuberculosis burden in the world, costing 170 million workdays to the country annually. But what is just as important as lost productivity now is lost potential in the future. It is becoming increasingly clear that on many measures of cognitive ability, malnourished Indian children perform two or three times worse than their adequately nourished peers. For an economy that will be more dependent on highly skilled workers, this poses a significant challenge. And it is one that really should be addressed given India's demographic outlook.

(UPSC CSAT 2023)

1. **Which one of the following statements best reflects what is implied by the passage?**

(*a*) Education system must be strengthened in rural areas.

(*b*) Large scale and effective implementation of skill development programmes is the need of the hour.

(*c*) For economic development, the health and nutrition of only skilled workers needs special attention.

(*d*) For rapid economic growth as envisaged by us, attention should be paid to the health and nutrition of the people.

Sol: **Option (a) is incorrect** although undernourished children are mentioned in the passage, rural areas are not particularly mentioned as the source of the issue. Although it doesn't immediately solve the current issue of how nutrition and health affect productivity, strengthening education can be beneficial in the future.

Option (b) is incorrect although the text highlights lost output due to health problems (such as anaemia and malnutrition), lack of skills is not the core focus of the passage. Rather loss of productivity due to malnutrition is the core focus.

Option (c) is incorrect because not only skilled workers, but the entire population of working age will be affected, as reflected in the first half of the passage. All-around economic growth would be impeded if the health of unskilled labourers is disregarded.

Option (d) is correct as directly reflects what is implied by the passage. It links the broader objective of economic growth to the detrimental effects of health problems on productivity. India can realise its economic goals and unleash the potential of its workforce by addressing issues related to health and nutrition.

Hence, option (d) is correct.

Passage

The majority of people who fail to accumulate money sufficient for their needs, are generally, easily influenced by the opinions of others. They permit the newspaper and the gossiping neighbours to do their thinking for them. Opinions are the cheapest commodities on the earth. Everyone has a flock of opinions ready to be wished upon by anyone who will accept them. If you are influenced by opinions when you reach decisions, you will not succeed in any undertaking. **(UPSC CSAT 2022)**

2. Which one of the following is implied by the passage?

(*a*) Most of the people do not accumulate money for their needs.

(*b*) Most of the people never fail to accumulate money for their needs.

(*c*) There are people who fail to accumulate money for their needs.

(*d*) There is no need to accumulate money.

Sol: **Option (*a*) is incorrect:** People want to accumulate money for their needs but they fail to do so because they rely on the opinions of others.

Option (*b*) is incorrect: We do not know what proportion of people fail to accumulate money for their needs.

Option (*d*) is incorrect: This option is beyond the scope of the passage. Passage nowhere explicitly or implicitly suggests that.

Option (*c*) is correct: From "The majority of people who fail to accumulate money sufficient for their needs" in the passage, this option can be easily implied.

Hence, option (c) is correct.

Passage

In the immediate future, we will see the increasing commodification of many new technologies artificial intelligence and robotics, 3D manufacturing, custom made biological and pharmaceutical products, lethal autonomous weapons and driverless cars. This will pose conundrums. The moral question of how a driverless car will decide between hitting a jaywalker and swerving and damaging the car has often been debated. The answer is both simple — save the human life — and complex. At which angle should the car swerve just enough to save the jaywalker or more than enough? If the driverless car is in Dublin, who would take the decision? The Irish Government, or the car's original code writer in California, or a software programmer in Hyderabad to whom maintenance is outsourced? If different national jurisdictions have different fine print on prioritising a human life, how will it affect insurance and investment decisions, including transnational ones? **(UPSC CSAT 2021)**

3. Which of the following statements best reflect the rational, plausible and practical implications that can be derived from the passage given above?

I. Too much globalization is not in the best interests of any country.

II. Modern technologies are increasingly blurring the economic borders.

III. Innovation and capital have impinged the domain of the State.

IV. Public policy of every country should focus on developing its own supply chains.

V. Geopolitics will have to reconcile to many ambiguities and Select uncertainties.

(*a*) I, IV and V only   (*b*) I, II, III and IV only

(*c*) II, III and V only   (*d*) I, II, III, IV and V

Sol: **Statement-I is incorrect** as it cannot be derived from the passage as it is nowhere mentioned explicitly that too much globalisation can hamper the best interest of any country.

Statement-II is correct because the passage revolves around the theme of transnational flow of modern technologies which is eliminating economic boundaries.

Statement-III is correct as capital and innovation have definitely affected the domain of the state which can be directly inferred from the passage.

Statement-IV is incorrect because it focuses on countries going for their own supply chains which might not be the case.

Statement-V is correct as it is a general statement which caters to the overall theme of the passage which is transnational understanding of human life with all the ambiguities and the uncertainties.

Hence, option (c) is correct.

5 Assumptions Based Reading Comprehension

INTRODUCTION

- There are two main types of these questions. The first type involves identifying assumptions made by the passage's author to communicate the central message or theme. In these questions, candidates analyse the passage to recognize the underlying beliefs or ideas shaping the author's perspective.

- The second type of assumption-based question provides statements and asks candidates to determine which ones can be assumed based on the message conveyed in the passage. Here, candidates need to carefully consider the information presented in the passage and infer implicit assumptions that support or relate to the statements given.

- In essence, assumption-based questions go beyond surface-level comprehension and demand a deeper examination of the thought process behind the passage. Candidates must showcase their aptitude for logical reasoning by identifying the implicit assumptions that underpin the ideas presented in the passage, demonstrating a comprehensive grasp of the content.

ASSOCIATED KEYWORDS

- The associated keywords present in this type of questions are "most logical assumption" "most valid assumption", "assumption underlying the message of passage", "assumption if true negates the central message".

- All the assumption based questions will have this word and this mentioned word allows you to go a little outside the related central theme of the passage. Because an assumption will not be directly mentioned in the passage and hence the options that are explicitly mentioned in the passage are incorrect.

NATURE OF ASSUMPTION BASED QUESTIONS

- These questions are a bit tough, unlike other questions that check you on implicit messages in the passage, assumption-based questions need you to think more deeply. They want you to figure out ideas that the writer doesn't directly talk about in the passage. Because of this, these questions are considered moderate to hard in difficulty.

- Answering them well means going beyond just knowing the main points of the passage. You need to notice the hidden assumptions, the things the writer believes without saying them out loud. So, these questions are like a puzzle that tests how well you can really understand and think about what you're reading.

Tips and Tricks:

- Avoid quickly scanning through the passage. Take your time to read the passage carefully and fully understand it.
- Identify and underline keywords in the passage, particularly phrases that capture the central theme. Assumptions are often related to this central theme.
- Focus on the concluding remarks of the passage. Authors might make assumptions while wrapping up, so pay close attention to these statements.
- While assumption questions encourage creative thinking, ensure that your assumptions stay within the passage's scope. They should be directly based on or challenge something explicitly mentioned in the passage.

Previous Year Questions

Directions for the following items: Read each of the following passages and answer the items that follow. Your answers of these items should be based on the passages only.

Passage

In India, the segregation of municipal waste at source is rare. Recycling is mostly with the informal sector. More than three-fourths of the municipal budget goes into collection and transportation, which leaves very little for processing/ resource recovery and disposal. Where does waste-to-energy fit into all this? Ideally it fits in the chain after segregation (between wet waste and rest), collection, recycling, and before getting to the landfill. Which technology is most appropriate in converting waste to energy depends on what is in the waste (that is biodegradable versus non-biodegradable component) and its calorific value. The biodegradable component of

India's municipal solid waste is a little over 50 per cent, and biomethanation offers a major solution for processing this.

1. **Based on the above passage, the following assumptions have been made:**

    I. Collection, processing and segregation of municipal waste should be with government agencies.

    II. Resource recovery and recycling require technological inputs that can be best handled by private sector enterprises.

    Which of the assumptions given above is/are correct?

    (*a*) I only
    (*b*) II only
    (*c*) Both I and II
    (*d*) Neither I nor II

Sol: **Assumption I is incorrect:** The passage mentions that 'More than three-fourths of the municipal budget goes into collection and transportation, which leaves very little for processing/resource recovery and disposal. Thus, this statement seems to assume the opposite of what the passage actually infers.

Assumption II is incorrect: Though the statement talks about technological intervention, it does not comment upon the efficiency of the private or public sector.

Hence, option (d) is correct.

Passage

In some places in the world, the productivity of staples such as rice and wheat has reached a plateau. Neither new strains nor fancy agrochemicals are raising the yields. Nor is there much unfarmed land left that is suitable to be brought under the plough. If global temperature continues to rise, some places will become unsuitable for farming. Application of technology can help overcome these problems. Agricultural technology is changing fast. Much of this change is brought about by affluent farmers in the West/ Americas. Techniques developed in the West are being adapted in some places to make tropical crops more productive. Technology is of little use if it is not adapted. In the developing world, that applies as much to existing farming techniques as it does to the latest advances in genetic modification. Extending to the smallholders and subsistence farmers of Africa and Asia the beat of today's agricultural practices, in such simple matters aa how much fertilisers to apply and when, would lead to a greatly increased availability of food for humanity. So would things like better roads and storage facilities, to allow for the carriage of surpluses to markets and reduce wastage.

2. **Based on the above passage, the following assumptions have been made:**

    I. Poor countries need to bring about change In their existing farming techniques.

    II. Developed countries have better infrastructure and they waste less food.

    Which of the above assumptions is/ are valid?

    (*a*) I only
    (*b*) II only
    (*c*) Both I and II
    (*d*) Neither I nor II

Sol: **Assumption I is valid:** From "Agricultural technology is changing fast." and "Technology is of little use if it is not adapted. In the developing world, that applies as much to existing farming techniques as it does to the latest advances in genetic modification," It is evident that this assumption is correct.

Assumption II is invalid: Though the passage suggests the need of better infrastructure for availability of sufficient food but We do not have enough information in the passage to assume that developed countries have better infrastructure.

Hence, option (a) is correct.

Passage

Fig trees (genus Ficus) are considered sacred in India, East Asia and Africa and are common in agricultural and urban landscapes where other large trees are absent. In natural forests, fig trees provide food for wildlife when other resources are scarce and support a high density and diversity of frugivores (fruit-eating animals). If frugivorous birds and bats continue to visit fig trees located in sites with high human disturbance, sacred fig trees may promote frugivore abundance. Under a favourable microclimate, plenty of seedlings of other tree species would grow around fig trees.

3. **On the basis of the passage given above, the following assumptions have been made:**

    I. Fig trees can often be keystone species in natural forests.

    II. Fig trees can grow where other large woody species cannot grow.

    III. Sacred trees can have a role in biodiversity conservation.

    IV. Fig trees have a role in the seed dispersal of other tree species.

    Which of the above assumptions is/are valid?

    (*a*) I and II only
    (*b*) III only
    (*c*) II and IV only
    (*d*) I, III and IV only

Sol: **Assumption I is valid:** It can be assumed based on phrases "provide food for wildlife," "support a high density and diversity of frugivores," and "plenty of seedlings of other tree species would grow around fig trees" in the passage.

Assumption II is invalid: Fig trees do grow where other large trees are absent but it can not be assumed that other species of trees can not grow there.

Assumption III is valid: It can be assumed based on phrases "support a high density and diversity of frugivores," "plenty of seedlings of other tree species would grow around fig trees" in the passage.

Assumption IV is valid: The last statement clearly states fig trees' property with respect to the seed dispersal.

Hence, option (d) is correct.

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Information Based Reading Comprehension

ASSOCIATED KEYWORDS

These appear to be simple reading comprehension question with keywords like "which of the following statements is correct/what author is trying to say/which policy or solution can be adopted based on the content of the passage" that ask about the main idea, the author's position, or practical solutions based on the text.

NATURE OF INFORMATION BASED QUESTIONS

- **Pay close attention to comprehension:** Although it may seem clear, complete understanding is crucial. As often as necessary, reread the passage and actively engage with its content. As you read, make note of possible responses, undeline important passages, and highlight transitions.

- **Be wary of options with tight wording:** Avoid choosing solutions that mimic the passage but fall short of the mark. Accurate comprehension of the author's meaning is essential. Don't omit the sentence that contains the solution; instead, focus on how it aligns with the author's intent and supports the main argument. Examine it in light of the passage's overall structure as well as the surrounding paragraph, considering the flow of ideas and context.

Tips and Tricks:

Spot the keyword: Look for keyword and phrases that signal the author's intent, such as "therefore," "consequently," or "in conclusion." These often point towards the main idea or solution.

PREVIOUS YEAR QUESTIONS

Directions for the following items: Read each of the following passages and answer the items that follow. Your answers of these items should be based on the passages only.

Passage

The last end of the state is not to dominate men, nor to restrain them by fear; rather it is so to free each man from fear that he may live and act with full security and without injury to himself or his neighbour. The end of the state, I repeat, is not to make rational beings into brute beasts and machines. It is to enable their bodies and their minds to function safely. It is to lead men to live by, and to exercise, a free reason; that they may not waste their strength in hatred, anger and guile, nor act unfairly toward one another." **(UPSC CSAT 2023)**

1. **Based on the above passage, which one of the following terms expresses the ultimate goal of the state?**

   (*a*) Personal safety   (*b*) Health of body and mind

   (*c*) Communal harmony   (*d*) Liberty

Sol: **Option (*a*) is incorrect** because the passage discusses safeguarding people from harm and fear, which enhances liberty but goes beyond simple personal safety. It highlights the unrestricted ability to live and act.

Option (*b*) is incorrect because the passage acknowledges the importance of physical and mental well-being, but its primary focus is not healthcare itself. It emphasises enabling individuals to use their faculties freely, not just ensuring their physical and mental health.

Option (*c*) is incorrect because while peaceful coexistence is desirable, the passage focuses on individual liberty, not specifically on interpersonal relationships. It emphasises freedom from fear and the ability to exercise reason, which benefits everyone but is not synonymous with communal harmony.

Option (*d*) is correct because the passage emphasises freedom from fear, restraint, and irrationality multiple times. It favours people living and acting in complete security, devoid of fear, and in line with their own logic. This is in perfect accordance with the fundamental idea of liberty.

Hence, option (d) is correct.

Passage

Unless the forces and tendencies which are responsible for destroying the country's environment are checked in the near future and afforestation of denuded areas is taken up on a massive scale, the harshness of the climatic conditions and soil erosion by wind and water will increase to such an extent that

agriculture, which is the mainstay of our people, will gradually become impossible. The desert countries of the world and our own desert areas in Rajasthan are a grim reminder of the consequences of large-scale deforestation. Pockets of desert-like landscape are now appearing in other parts of the country including the Sutlej-Ganga Plains and the Deccan Plateau. Where only a few decades back there used to be lush green forests with perennial streams and springs, there is only brown earth, bare of vegetation, without any water in the streams and springs except in the rainy season.   **(UPSC CSAT 2022)**

2. **According to the passage given above, deforestation and denudation will ultimately lead to which of the following?**

   I.   Depletion of soil resource

   II. Shortage of land for the common man

   III. Lack of water for cultivation

   Select the correct answer using the code given below.

   (*a*) I and II only   (*b*) II and III only

   (*c*) I and III only   (*d*) I, II and III

Sol: **Statements I and III are correct:** The passage mentions that the Pockets of desert-like landscape are now appearing in other parts of the country including the Sutlej-Ganga Plains and the Deccan Plateau means there would be lack of water for cultivation.Deforestation would eventually deplete the soil resource.

Statement-II is incorrect since we do not have enough information in the passage to determine whether there will be shortage of land or not.

Hence, option (c) is correct.

Passage

The resolution of bankruptcy cases of Indian banks under the Insolvency and Bankruptcy Code should help bring non-performing assets (NPA) situations under some control. Despite the slow pace of resolutions by the National Company Law Tribunal, the Code can be helpful in cleaning up bank books in future credit cycles. The recapitalisation of public sector banks too can help increase the capital cushion of banks and induce them to lend more and boost economic activity. But bad debt resolution and recapitalisation are only a part of the solution as they, by themselves, can do very little to rein in reckless lending that has pushed the Indian banking system to its current sorry state. Unless there are systemic reforms that address the problem of unsustainable lending, future credit cycles will continue to stress the banking system.   **(UPSC CSAT 2021)**

3. **Which one of the following statements best reflects the most logical, rational and practical suggestion implied by the passage given above?**

   (*a*) Lending by the banks should be closely monitored and regulated by the Central Government.

   (*b*) Interest rates should be kept low so as to induce banks to lend more, promote credit growth and thereby boost economic activity.

   (*c*) Merger of many banks into a few large banks alone is the long-term solution to make them viable and prevent their bad performance.

   (*d*) Indian banking system requires structural reforms as a long-term solution for bad loans problem.

Sol: **Option (*a*) is incorrect** as it is a suggestion or advice that states central government monitoring but it is nowhere mentioned in the passage.

Option (*b*) is incorrect as it is nowhere mentioned in the passage and cannot be inferred either.

Option (*c*) is incorrect as it is also not mentioned anywhere in the whole passage which talks about bank mergers.

Option (*d*) is correct based on the statement "Unless there are systemic reforms that address the problem of unsustainable lending, future credit cycles will continue to stress the banking system" given in the passage.

Hence, option (d) is correct.

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